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Minimization of MachineVibrations

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Preface

The presented textbook was developed by way of cooperation between the Technical University in Liberec and the HochschuleZittau/Görlitz.

The textbook supports the training module

„MinimizationofMachineVibrations“

and presents the selected chapters from the teaching of engineering subjects at both partner universities. The Czech version of this publication is identical in content to the version written in the German language and is intended for further education, in particular of university students. Content-related and formally educational topics related to the current needs for technical practice cut across the present publication.

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Content

1 Fundamentals of Machine Dynamics

Each machine is a dynamic system that consists of machine elements. According to their functions, these elements are mounted to the frame, rack, cabinet or to each other in a fixed, movable or flexible manner. During operation, the power from the engine is transmitted through the transmission mechanism to the working mechanisms. Thanks to power transmission and movement of the components of this chain, dynamic forces and moments are formed as well as vibrations of the components and their frame.

The intensity of vibration of machine elements, frames, or the entire machines does not only depend on power excitation but also on dynamic parameters of the complex system. Therefore, the correct design and dimensioning of the parts are very important in order to reduce oscillations and vibrations of the machine and its surroundings. Optimal setting of the dynamic parameters is one of the basic tasks of the designer.

1.1 Dynamic Parameters

Dynamic parameters of the system, which can be the machine or any other device, significantly decide on its behaviour during operation. In relation to the nature of the excitation force effects, the kinematics of dynamic systems is mainly affected by the magnitude of mass and inertia effects of the components, the modulus of rigidity of springs and other elastic elements, damping of movable joints, or special components used for the purpose. Knowledge of the values of dynamic parameters is necessary to build and solve a computational model of the system, on the basis of which the operating behaviour of the system can be optimised from different perspectives. Some of the methods for determining dynamic parameters will be described in the following paragraphs.

1.1.1 Mass and Inertia Parameters

The size of the mass of movable and stationary machine elements or the entire machines can be usually easily found or can be obtained from the manufacturer's documentation. The computational models also require necessary knowledge of the position of the centre of gravity and inertial effects of the individual components or the entire machine. In some cases, this information can be estimated or obtained from 3D models, but mostly they are found by measurements, thus achieving higher accuracy. An approximate calculation of the mass and inertia parameters is used only when the component or the entire machine consists of relatively simple geometric shapes.

Experimental investigation of the position of the centre of gravity of objects (body, component, machine, etc.) with at least one plane of symmetry is explained in *Fig. 1*.1.

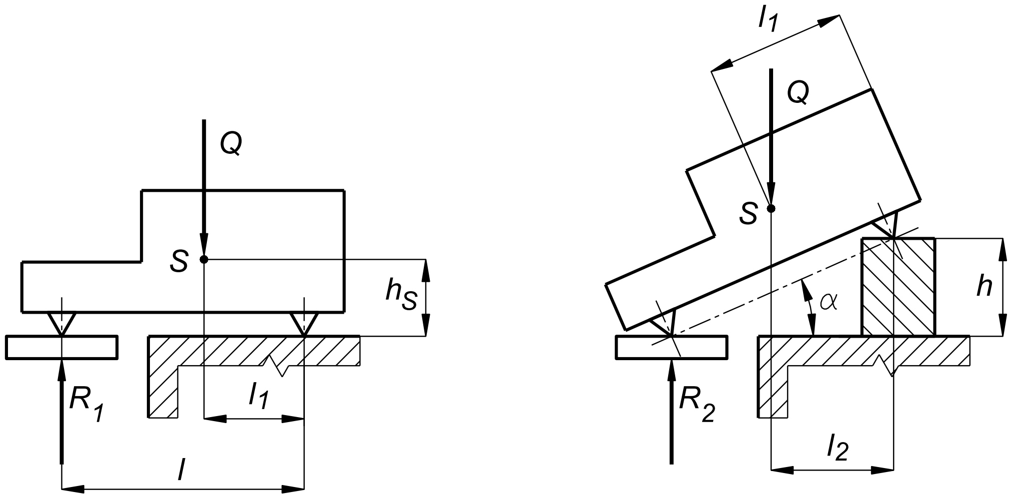


Fig. 1.1 Determining the centre of gravity

Based on the findings of equations R_1 and R_2 after double weighing,

$$l_1 = \frac{R_1 l}{Q}, \tag{1.1}$$

and

$$l_2 = \frac{R_2 l cos\alpha}{Q}, \tag{1.2}$$

where

$$\alpha = arcsin\frac{h}{l}. \tag{1.3}$$

Further,

$$h_s = \left(\frac{l_2}{cos\alpha} - l_1\right)\frac{l}{tg\alpha}. \tag{1.4}$$

After substitution, the position h_s of the centre of gravity can be determined. Then,

$$h_s = \frac{R_2 - R_1}{Q}\frac{l^2}{h}\sqrt{1 - \left(\frac{h}{l}\right)^2}. \tag{1.5}$$

The accuracy of the results obtained depends on the difference between the measured responses R_1 and R_2, where appropriate, on the height h of the pedestal.

Dynamic calculation of the system with a rotary motion requires knowledge of principal axes of inertia and moments of inertia around these axes. Such parameters are most often determined with the use of torsional vibration of an object to be examined on the defined hinge.

The principle of this method is that in the Cartesian coordinate systemx, y, z it is possible to define three moments of inertiaJ_x, J_y and J_z, and three products of inertiaD_{xy}, D_{xz} and D_{yz}

$$J_x = \int_m (y^2 + z^2)dm, \tag{1.6}$$

$$J_y = \int_m (x^2 + z^2)dm, \tag{1.7}$$

$$J_z = \int_m (x^2 + y^2)dm, \tag{1.8}$$

$$D_{xy} = -\int_m xydm, \tag{1.9}$$

$$D_{xz} = -\int_m xzdm \tag{1.10}$$

and

$$D_{yz} = -\int_m yzdm. \tag{1.11}$$

For any axis$a$, which passes through the origin 0 of the coordinate system and forms with its axes x, y, z angles α, β, γ,, the moment of inertia is calculated according to the relation

$$\begin{aligned} J_a &= J_x cos^2\alpha + J_y cos^2\beta + J_z cos^2\gamma + 2D_{xy} cos\alpha cos\beta + \\ &+ 2D_{xz} cos\alpha cos\gamma + 2D_{yz} cos\beta cos\gamma. \end{aligned} \tag{1.12}$$

If we substitute the results of the relation (1.12) for various axes$a$, or angles α, β, γ, into the equation

$$r_A = \frac{C}{\sqrt{J_a}}, \tag{1.13}$$

we obtain the surface where r_A is the distance of a point A on the axis a from the origin 0 of the coordinate system, $C(\sqrt{kgm^2})$ while being an arbitrary constant.

The coordinates of the point A of this surface are

$$x_A = r_A cos\alpha = \frac{C}{\sqrt{J_a}} cos\alpha, \tag{1.14}$$

$$y_A = r_A cos\beta = \frac{C}{\sqrt{J_a}} cos\beta, \tag{1.15}$$

and

$$z_A = r_A cos\gamma = \frac{C}{\sqrt{J_a}} cos\gamma. \tag{1.16}$$

After substitution of (1.14), (1.15) and (1.16) into the relation (1.12),

$$J_x x_A^2 + J_y y_A^2 + J_z z_A^2 + 2D_{xy} x_A^2 y_A^2 + 2D_{xz} x_A^2 z_A^2 + 2D_{yz} y_A^2 z_A^2 = C^2. \tag{1.17}$$

The expression (1.17) represents an ellipsoid in the analytic geometry. If the origin 0 of the coordinate system coincides exactly with the centre of gravity S of an object, it is the equation of an ellipsoid of inertia. When the coordinates x, y, z are positioned so that the products of inertia D_{xy}, D_{xz}, D_{yz} are equal to zero, the respective axes are regarded as the principal axes of inertia.

If the object to be examined has one plane of symmetry, two of the principal axes of inertia can be found in this plane and in the relation (1.17), one coordinate is zero. Then,

$$J_x x_A^2 + J_z z_A^2 + 2D_{xz} x_A z_A = C^2 \tag{1.18}$$

and the spatial problem is reduced to the plane role.

The equation (1.18) corresponds to the ellipse of inertia, which can be determined by three points. These three points are obtained by measuring the moments of inertia around the three axes.

The object shown in *Fig. 1.2* can be mounted on the torsion bar with known torsional rigidity c_t and allowed to torsionally vibrate. Using this test in the three axes we can obtain the values of the moments of inertia J_1,J_2 and J_3 around the axes a_1, a_2 and a_3 in the plane of symmetry (*Fig. 1.2*). It is preferred to select two axes perpendicular to each other and thereby establish a coordinate system x, y, z.

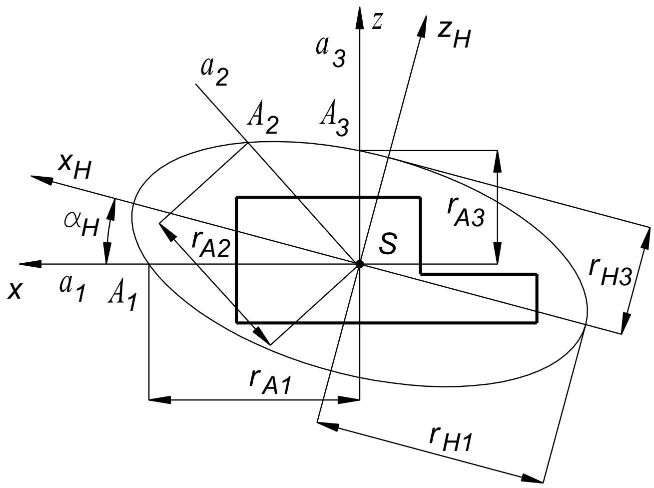


Fig. 1.2 Determination of the main moments of inertia and their axes

The moments of inertia J_{a1}, J_{a2} and J_{a3} are obtained from the oscillation periods T_1, T_2, T_3 of an object and the known torsional rigidity k_t of the torsion bar. Then

$$J_{a1} = \frac{T_1^2}{4\pi^2} c_t, \tag{1.19}$$

$$J_{a2} = \frac{T_2^2}{4\pi^2} c_t \tag{1.20}$$

and

$$J_{a3} = \frac{T_3^2}{4\pi^2} c_t. \tag{1.21}$$

The points A_1, A_2, A_3 of the ellipse on the axes a_1, a_2 and a_3 have the vectors r_{A1}, r_{A2}, r_{A3}, the values of which at arbitrary constant C are calculated from the following relations:

$$r_{A1} = \frac{C}{\sqrt{J_{a1}}}, \tag{1.22}$$

$$r_{A2} = \frac{C}{\sqrt{J_{a2}}} \tag{1.23}$$

and

$$r_{A3} = \frac{C}{\sqrt{J_{a3}}}. \tag{1.24}$$

After substitution of the known coordinates $x_{A1}, z_{A1}, x_{A2}, z_{A2}, x_{A3}, z_{A3}$ into the central ellipse equation

$$a_{11}x^2 + a_{12}xz + a_{22}z^2 = 1 \tag{1.25}$$

we obtain the system of linear equations for the coefficients a_{11}, a_{12} and a_{22}:

$$a_{11}x_{A1}^2 + a_{12}x_{A1}z_{A1} + a_{22}z_{A1}^2 = 1, \tag{1.26}$$

$$a_{11}x_{A2}^2 + a_{12}x_{A2}z_{A2} + a_{22}z_{A2}^2 = 1, \tag{1.27}$$

$$a_{11}x_{A3}^2 + a_{12}x_{A3}z_{A3} + a_{22}z_{A3}^2 = 1. \tag{1.28}$$

The position of the principal axes of inertia x_H and z_H is defined by an angle α_H. Then,

$$x_{H1} = r_{H1}cos\alpha_{H1} \tag{1.29}$$

and

$$z_{H1} = r_{H1}sin\alpha_{H1}. \tag{1.30}$$

After substituting the relations (1.29) and (1.30) into the equations (1.26),

$$a_{11}cos^2\alpha_{H1} + 2a_{12}cos\alpha_{H1}sin\alpha_{H1} + a_{22}sin^2\alpha_{H1} = \frac{l}{r_{H1}^2}. \tag{1.31}$$

Since the vector r_{H1} is an extreme value, the angle α_{H1} is based on the condition that the derivative of a function

$$\frac{l}{r^2} = a_{11}cos^2\alpha + 2a_{12}cos\alpha sin\alpha + a_{22}sin^2\alpha \tag{1.32}$$

according to α is equal to zero

$$\frac{d\left(\frac{l}{r^2}\right)}{d\alpha} = 0. \tag{1.33}$$

After adjustment

$$\alpha_{H1} = \frac{1}{2}arctg\frac{-2a_{12}}{a_{22} - a_{11}}. \tag{1.34}$$

The values r_{H1} and r_{H3} can be calculated from the relations

$$r_{H1} = \sqrt{\frac{1}{a_{11}cos^2\alpha_{H1} + 2a_{12}cos\alpha_{H1}sin\alpha_{H1} + a_{22}sin^2\alpha_{H1}}} \tag{1.35}$$

and

$$r_{H3} = \sqrt{\frac{1}{a_{11}cos^2\alpha_{H3} + 2a_{12}cos\alpha_{H3}sin\alpha_{H3} + a_{22}sin^2\alpha_{H3}}}, \tag{1.36}$$

where

$$\alpha_{H3} = \alpha_H + \frac{\pi}{2}. \tag{1.37}$$

Then you can calculate the appropriate principal moments of inertia J_1 and J_3 from the relations

$$J_1 = \frac{1}{r_{H1}^2} \tag{1.38}$$

and

$$J_3 = \frac{1}{r_{H3}^2}. \tag{1.39}$$

The procedure is clear; however, it is applicable only to objects with at least one plane of symmetry. In general, this task is correspondingly more complicated. But you can find this solution in the literature. There are also computational programs that are able to tackle such tasks.

1.1.2 Parameters of Springs

Springs immediately affect the behaviour of the mechanical system during linear or rotational motion of an object. Their springing characteristics are distinguished by the type of motion of an oscillating object and the longitudinal stiffness c during translational motion and torsional stiffness c_t during rotation.

Steel springs often have a linear characteristic (dependence of restoring force F_R on deformation z), corresponding to a constant stiffness. For this reason, steel springs cause relatively small problems in the calculations of dynamic systems. This is different for systems containing rubber or air springs, which are characterised by non-linear characteristics (*Fig. 1*.3), which are often hysteresis loaded. Hysteresis pertains to the internal damping of springs and depends on velocity. Exact solution to the oscillatory motion is difficult in such cases and can only be executed numerically.

Generally, there are efforts made to simplify the calculation by linearising the characteristics of rubber and air springs near the working position and assuming small amplitudes. Then, a constant stiffness can be substituted in the computational model (*Fig. 1*.4). Non-linear characteristics should be considered only at larger amplitudes of oscillatory motion, which, however, often uses a guiding mechanism for the motion of an object, which reduces the number of degrees of freedom. It can therefore be concluded that non-linear parameters of springs are considered in the case of oscillatory motion in one or two coordinates.

Machine elements that are not springs also have springing characteristics. Examples include: gears, belts, chains, ropes, clutches, levers, shafts, etc. All these components are used in the transmissions in rotary motion and bring elastic couplings with torsional stiffness into the system. They depend on various design parameters and are generally only roughly estimated. In such cases, the relevant measurement is recommended for more precise definition of the springing characteristics.

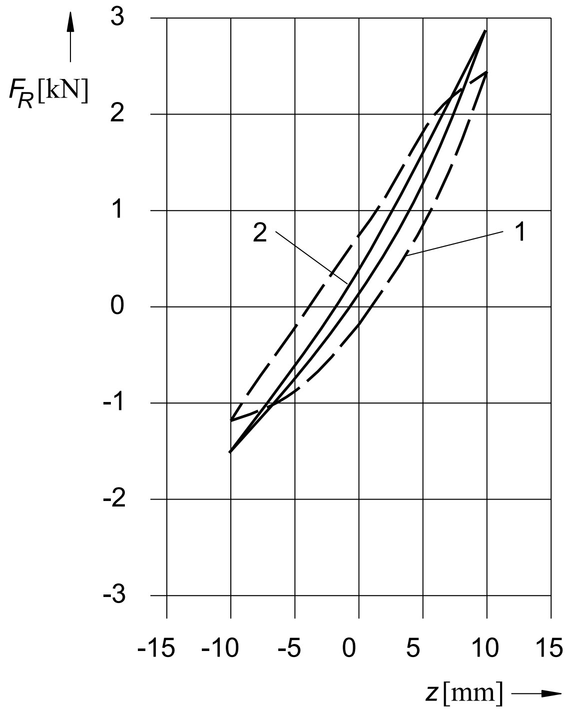


Fig. 1.3 *Characteristics of air springs at an amplitude of 10 mm:*
1- $\Omega = 10\,\mathrm{rad\,s^{-1}}$, *2-* $\Omega = 100\,\mathrm{rad\,s^{-1}}$

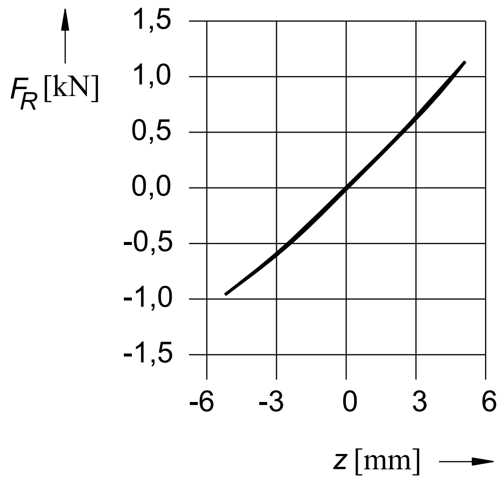


Fig. 1.4 *Characteristics of air springs at an amplitude of* 5 mm *and* $\Omega = 10\,\mathrm{Hz}$

The springing characteristics in transmission gears are determined by the parameters of gearing and the width of meshing gears (*Fig. 1*.5). However, mathematical description of mesh ratios is generally very difficult. Based on the non-integer coefficient of mesh, the number of meshing teeth changes, and thus the torsional rigidity c_{t1} and c_{t2} of gears pulses, resulting in a parametric excitation during operation. In dynamic solution, the mean value c_v of stiffness during one period is usually considered.

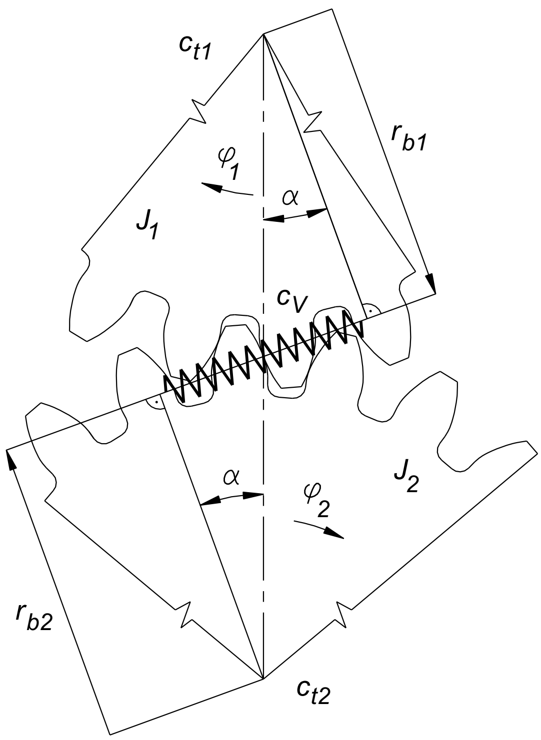


Fig. 1.5 Elastic coupling of gears

Recommendations to determine the parameters of the elastic coupling of gears for specific pairs can be found in the literature. It turns out, however, that the values of these parameters are in most practical cases much more greater than the torsional rigidity of the respective shafts. If the value of the stiffness c_V of the elastic coupling of meshing gears is known, the equations of motion of the system can be written in the form of

$$J_1\ddot{\varphi}_1 + c_v r_{b1}(r_{b1}\varphi_1 + r_{b2}\varphi_2) = 0, \tag{1.40}$$

$$J_2\ddot{\varphi}_2 + c_v r_{b2}(r_{b2}\varphi_1 + r_{b2}\varphi_2) = 0, \tag{1.41}$$

where J_1 and J_2 are the moments of inertia, φ_1 and φ_2 the turning of shafts, r_{b1} and r_{b2} the radii of the base circles.

The equations (1.40) and (1.41) can be adjusted as follows:

$$J_1\ddot{\varphi}_1 + c_v\left(\frac{mz_1}{2}cos\alpha\right)^2\left(\varphi_1 + \frac{z_2}{z_1}\varphi_2\right) = 0, \tag{1.42}$$

$$J_2\ddot{\varphi}_2 + c_v\left(\frac{mz_2}{2}cos\alpha\right)^2\left(\varphi_2 + \frac{z_1}{z_2}\varphi_1\right) = 0. \tag{1.43}$$

Torsional rigidity c_{t1} and c_{t2} is dependent on the module m of gearing, on the number of teeth z_1 and z_2, and on the pressure angle α.

For belt transmissions, torsional rigidity c_{t1} and c_{t2} depends on the diameters of pulleys and the rigidity c_R of the belt (*Fig. 1.*6).

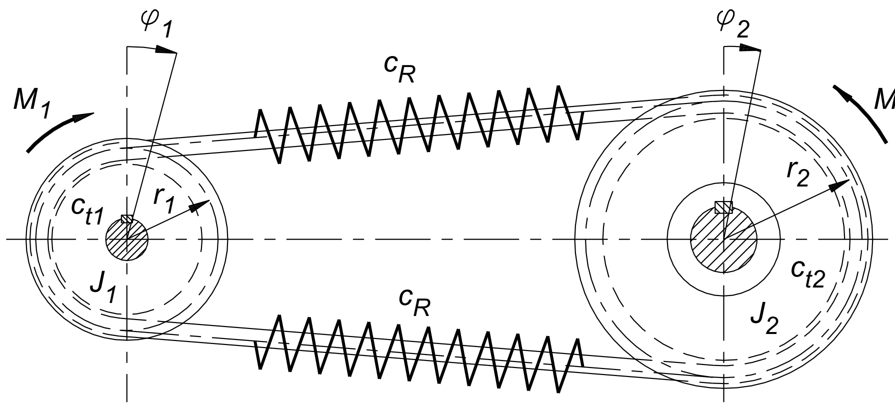


Fig. 1.6 Torsional rigidity of pulleys

In some cases, the parameters of the belt change with its load, and therefore it is necessary to know also transmission power ratios for the precise calculation of the motion. But mostly just put into the equations of motion for rotation φ_1 and φ_2 only one value of stiffness c_R of the belt. For stationary operation, the transmission will be

$$J_1\ddot{\varphi}_1 + 2c_R r_1(r_1\varphi_1 - r_2\varphi_2) = 0, \tag{1.44}$$

$$J_2\ddot{\varphi}_2 + 2c_R r_2(r_2\varphi_2 - r_1\varphi_1) = 0. \tag{1.45}$$

Torsional stiffness c_{t1} and c_{t2} are therefore dependent on radii r_1 and r_2 of pulleys and, like gears, on the gear ratio.

1.1.3 Damping Parameters

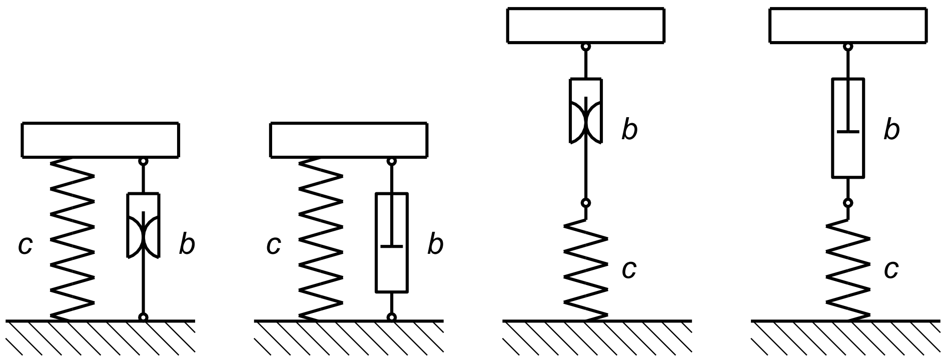
Partial dissipation of energy is always present in real mechanical systems with springing and damping elements. Physically it is the conversion of mechanical energy into heat. This process occurs by external influences such as friction forces and moments in bearings and guides, or forces of resistance during movement of the body in a liquid or gas. Furthermore, the damping can be observed as a result of internal friction in the material of the spring element. This effect is particularly noticeable at larger deformations of the elastic elements. There is hysteresis in load characteristic as with pneumatic springs at large deformations (*Fig. 1*.3).

Two types of external damping can be basically found in the mechanical systems. The first group exists independently of the will of the designer as a result of physical friction in bearings and in guides. The second group of external damping enters the system specifically via dampers, which operate on the basis of the viscosity of gases and liquids, and allow intensive influence or control of the oscillation of mechanical systems. Parameters of both kinds of damping can be included into the computational model for motion or vibration of the system.

The forces of resistance of damping act generally in parallel to the restoring forces of springs. The *Coulomb* damping model corresponds to the friction in movable joints of oscillating objects with frame (*Fig. 1*.7a). The *Kelvin* model describes damping forces that are based on the viscosity of a fluid (*Fig. 1*.7b). In addition, there are models in which the spring and dampers are arranged in sequence, in series, and transfer the same force or moment (*Fig. 1*.7c, d).

Damping of the dynamic system due to friction in bearings is always present. Its effect on the operation of the machine is usually negative, on the other hand, the use of damper with low friction forces in dynamic systems is a simple solution with low production costs.

The friction forces of resistance change abruptly their meaning according to the direction of relative velocity of the jointed parts.



a) b) c) d)

Fig. 1.7 *Mechanical models with dampers:*
a,c) friction damper, b,d) viscous damper

Damping force F_D of a friction damper can be expressed as

$$F_D = b\frac{\dot{z}}{|\dot{z}|}, \tag{1.46}$$

where b is the damping coefficient and $\dot{z}$ the velocity of vibratory motion. Substitution of the relation (1.46) into a computational model of a dynamic system leads to non-linear role.

Viscous damping causes forces of resistance due to motion of a body in a fluid. Dampers designed on this basis are very often used in different mechanical systems. The most famous use of these elements is in vehicles.

Damping force F_D of a viscous damper is introduced into the computational model through the relation

$$F_D = b\dot{z}. \tag{1.47}$$

With relatively small amplitudes of oscillatory motion of an object, the damping coefficient b is constant.

Its size is usually determined experimentally. In free vibration, two successive amplitudes z_{0k} and z_{0k+2} are measured in one direction (*Fig. 1*.8). Then, the logarithmic decrement Λ is determined according to the relation

$$\Lambda = ln\frac{z_{0k}}{z_{0k+2}}. \tag{1.48}$$

Also,

$$\Lambda = \frac{\pi b}{\sqrt{km}}, \tag{1.49}$$

where c is the spring stiffness and m the mass of an oscillating object.

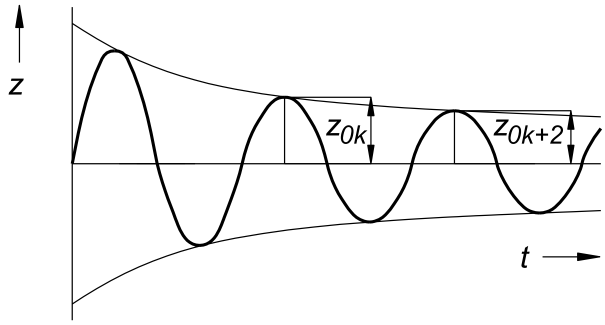


Fig. 1.8 *Determination of the damping coefficient using the logarithmic decrement*

For damping coefficient b,

$$b = \frac{\Lambda\sqrt{km}}{\pi}. \tag{1.50}$$

This procedure cannot be used in the dynamic systems with relatively large amplitudes of the oscillating motion. The computational model then represents a non-linear role and the damping force F_D should be substituted in the form of the polynomial obtained by measurement. Another solution to the computational model is made using some of the numerical methods.

To linearise the computational model with friction forces, the equivalent damping coefficient b_e is determined according to the criterion that the lost energy of the friction damper is the same as with the viscous damper with damping coefficient b_e. The energy equation will apply to one period

$$\pi b_e \omega z_0^2 = 4T z_0, \tag{1.51}$$

where is ω the circular frequency of oscillation, z_0 the amplitude and T the friction force.

For damping coefficient b_e,

$$b_e = \frac{4T}{\pi \omega z_0}. \tag{1.52}$$

1.2 Excitation Parameters

Causes, which bring the dynamic system into oscillatory motion, are referred to as excitation. They may take the form of force or kinematic.

Excitation can be deterministic or stochastic. Deterministic excitation is divided into periodic and others, e.g. impact. Stochastic excitations occur frequently as kinematic, for example in vehicles.

The determination of excitation parameters will be shown here only for the most common cases of practice, i.e. for periodic excitations. For example, torsional vibrations are thus generated by the action of periodic forces of burning gas in an combustion engine or bending vibrations in transverse load of rotating components. Oscillation of shafts may also arise as a result of periodic motion of the frame.

Impact force excitation occurs, for example, in presses and forging hammers, where they are the free vibrations of the dynamic system after impact. Initial conditions can be determined from the energy equations.

1.2.1 Periodic Excitation

Periodic excitations are introduced into the dynamic analysis usually in the form of a *Fourier* series of harmonic excitations with the same fundamental frequency Ω (*Fig. 1*.9). The mathematical procedure for substituting periodic excitation with a harmonic series is based on the equation

$$f(\Omega t) = a_0 + \sum_{k=1}^{\infty} a_k cos(k\Omega) + \sum_{k=1}^{\infty} b_k sin(k\Omega). \tag{1.53}$$

Individual harmonic functions are called in short as "harmonics" of the respective order k.

In practice, periodic excitation functions $f(\Omega t)$ are often obtained by measurement. There are measuring instruments, the so-called FFT-(*Fast Fourier Transformation)* analysers, which during measurement can determine the coefficients a_0, a_k and b_k. The amplitudes, which are shown in the frequency spectrum, are thus known (*Fig. 1*.10).

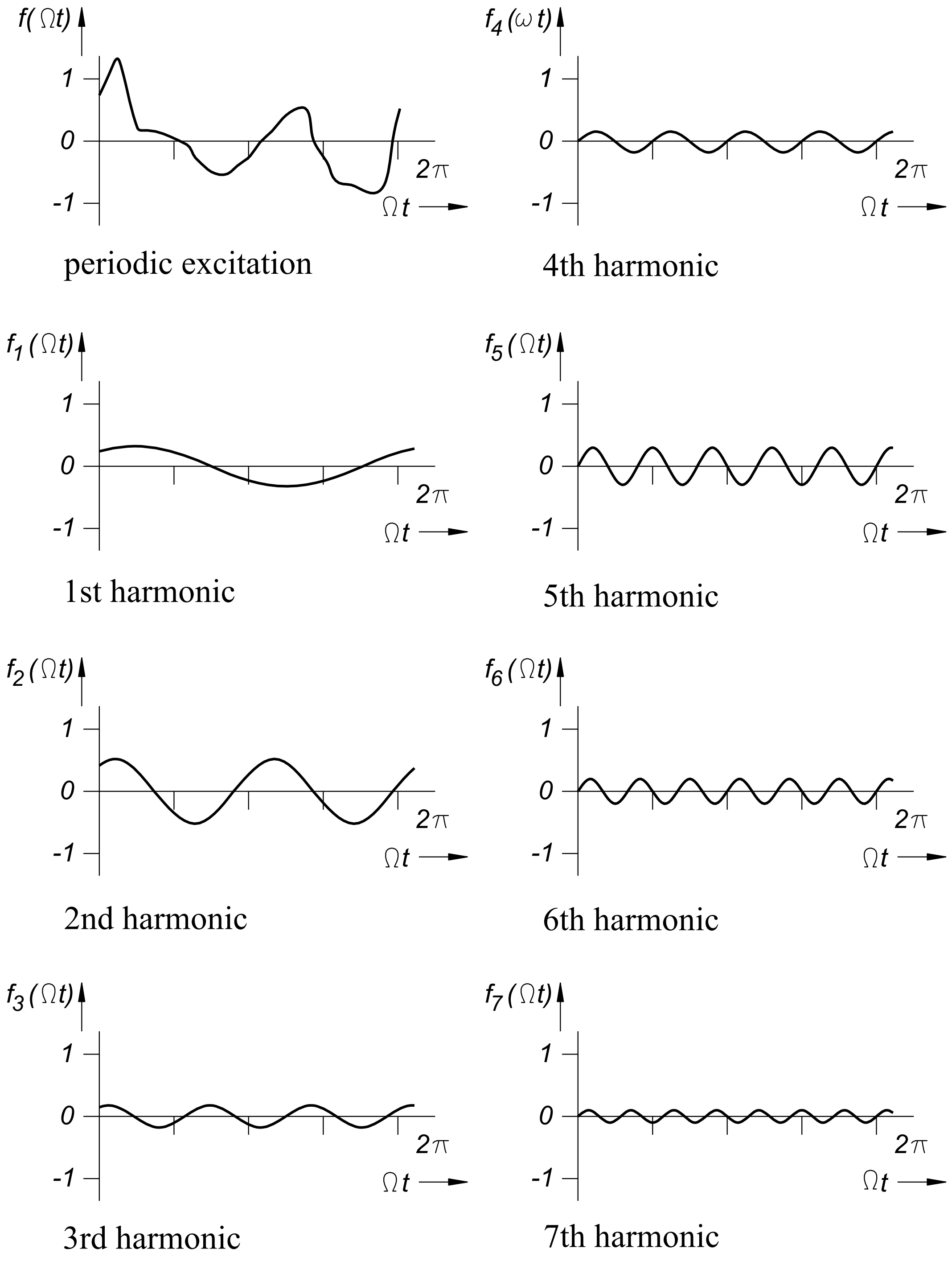


Fig. 1.9 Harmonics of the periodic excitation

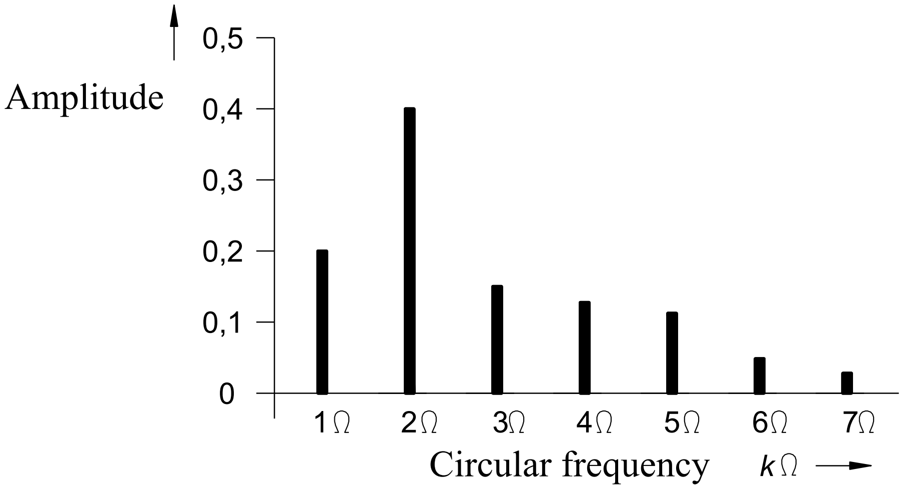


Fig. 1.10 *Frequency spectrum of the periodic excitation*

1.2.2 Non-periodic Excitations

Non-periodic excitations often occur during transient conditions. These are the startup or braking of machines, connection of aggregates, etc. Generally, the cases where the force action changes over time. In addition, there are machines, whose technological processes periodically generate impact excitation.

According to the time characteristic, two types of cases can be basically found in practice, namely the starting and impact excitation functions (*Fig. 1*.11).

Starting function is the change in load, relatively short, in time Δt. A typical example is the braking and the change in braking torque M_B (*Fig. 1*.11a).

The impact load function occurs for example in forging hammers or fast presses. In a short time, Δt the load increases from zero to maximum F_B and then again drops to zero (*Fig. 1*.11b). Meanwhile, the kinematic state of an impacted object changes.

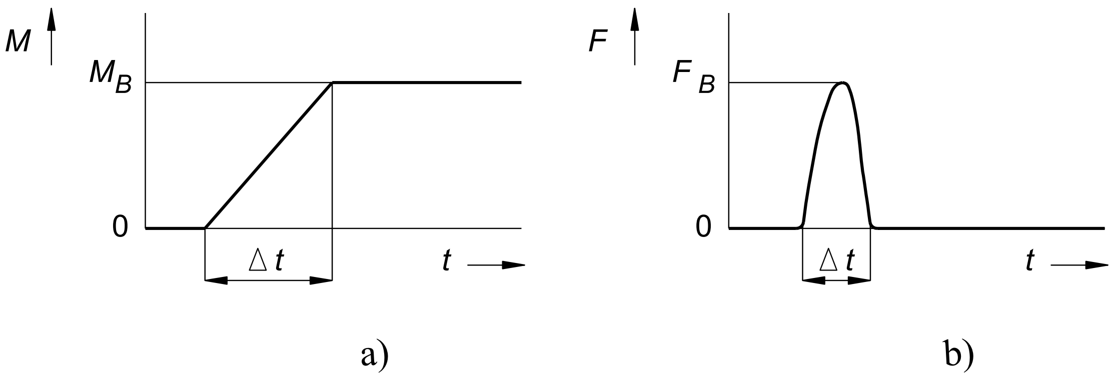


Fig. 1.11 *Non-periodic excitation: a) ramp function, b) impact function*

1.3 Computational Models

Behaviour of dynamic systems in the dynamics of machines is described using computational models. In the simplest case, it is a rigid body, which is connected to a base or a frame by a spring and possibly also a damper.

In general, this system has six degrees of freedom in the space. However, this number is very often reduced, with respect to the operating conditions, in the computational model up to a single degree of freedom. Such models include systems with a guide mechanism.

A dynamic system with one degree of freedom allows to relatively easily explain the link between excitation, dynamic parameters and kinematic quantities of the motion of an object. These principles can be applied even to more complicated cases.

1.3.1 Single-mass System with One Degree of Freedom

An oscillating dynamic system is an object with one degree of freedom, which is connected to a frame by means of springs and possibly also dampers, and carries out translational (*Fig. 1*.12) or rotary (*Fig. 1*.13) motion. Its location is accurately described using one coordinate. The coordinate is indicated as z and φ in translational motion and rotation, respectively.

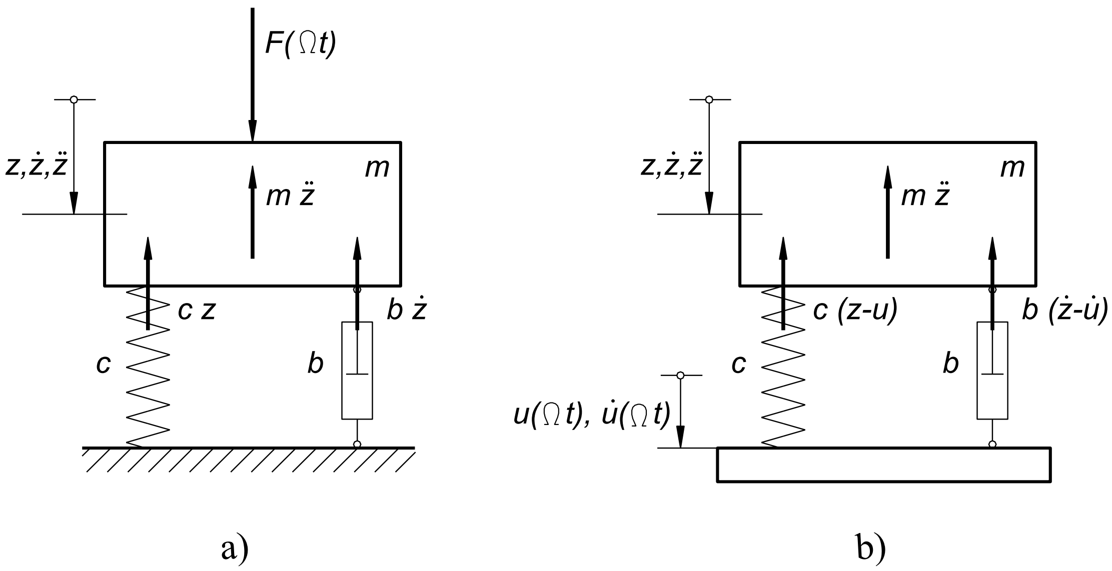


Fig. 1.12 Oscillatory system with translational movement and one degree of freedom:
a) force excitation, b) kinematic excitation

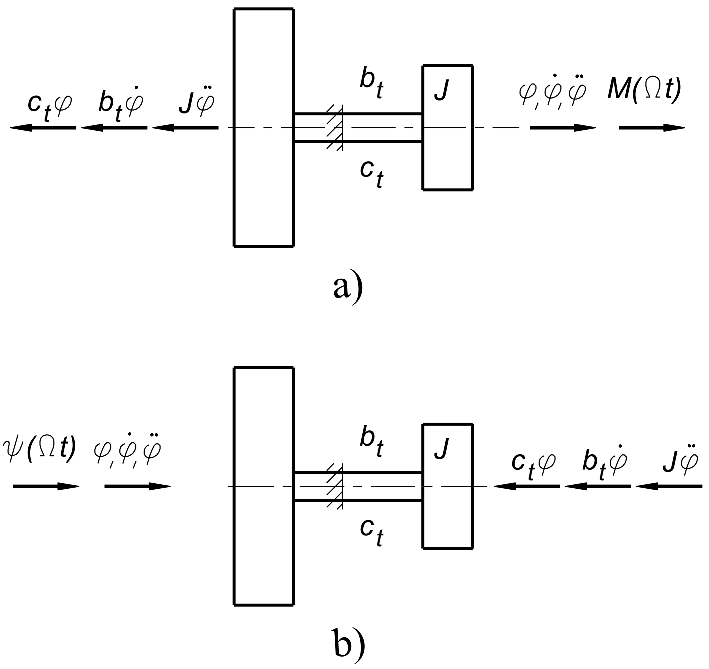


Fig. 1.13 Oscillatory system with rotational movement and one degree of freedom:
a) moment excitation, b) kinematic excitation

Vibration analysis will be performed for the system with a translational motion of an object (*Fig. 1*.12). The results also apply to the rotary motion, when the coordinate z is exchanged with the coordinate φ, the mass m with the moment of inertia J of an oscillating object to the axis of rotation, the stiffness k for the torsional rigidity c_t, the damping coefficient b for the torsional coefficient of damping b_t, and the excitation force F for the exciting moment M (*Fig. 1*.13).

A displacement oscillating system with one degree of freedom can be excited by a force F or a motion u of a frame or a base (*Fig. 1*.12). Both options may also occur simultaneously in practice. For linear systems, motions from different excitations can be added together. Both options may also occur simultaneously in practice. For linear systems, motions from different excitations can be added together.

In the analysis of the oscillatory motion, it is assumed that the stiffness of the spring c as well as the damping coefficient b are constant in the whole area of displacement z as well as the velocity $\dot{z}$. It is then a linear system.

System behaviour according to *Fig. 1*.12 and without excitation F (free oscillation) can be described by a linear differential equation of 2nd order, for example, based on the *d'Alembert's* principle:

$$m\ddot{z} + b\dot{z} + cz = 0. \tag{1.54}$$

Free vibrations occur when an object is deflected from the equilibrium position and reaches a deflection z_0 and, for example, initial velocity $\dot{z}_0$ and then is left to itself.

After the fundamental substitution $z = Ce^{\lambda t}$, the equation (1.54) is obtained for the coordinate z of free oscillations of the body and mass m:

$$z = z_0 e^{-\delta t}\left[\frac{\delta}{\omega} sin\omega t + cos\omega t\right] + \frac{\dot{z}_0}{\omega} e^{-\delta t} sin\omega t, \tag{1.55}$$

where the deflection z_0 and the velocity $\dot{z}_0$ represent the values of deflection and velocity in the initial state of an oscillating system.

The values

$$\omega = \sqrt{\frac{c}{m} - \frac{b^2}{4m^2}} \tag{1.56}$$

and

$$\delta = \frac{b}{2m} \tag{1.57}$$

indicate the natural frequency of free oscillations and the attenuation (damping) of the amplitude.

The natural frequency ω of free undamped oscillations is

$$\omega = \sqrt{\frac{c}{m}} = \omega_0. \tag{1.58}$$

Thus, a damped system always has lower natural frequency than a system without damping.

The relation (1.56) shows that with the damping coefficient

$$b_{kr} = 2\sqrt{cm} \tag{1.59}$$

the natural frequency is $\omega = 0$. In this case, it is the aperiodic motion. The damping coefficient (1.59) is referred to as the critical damping b_{kr}.

If the dynamic system is excited by force or kinematics, the object performs forced oscillations. Excitation may be periodic or aperiodic.

Kinematic quantities of forced oscillations of an object that are excited by force can be calculated from the following differential equation:

$$m\ddot{z} + b\ddot{z} + cz = F, \tag{1.60}$$

where

$$F = F_0 sin\Omega t. \tag{1.61}$$

In relation (1.61), the force amplitude and the excitation frequency are referred to as F_0 and Ω, respectively.

The reason that the excitation force F is introduced as a harmonic is in the possibility of substituting any periodic excitation with a *Fourier* series of harmonic functions. Their frequencies are relative to the fundamental frequency in the integer ratio.

The solution to the equation (1.60) contains free and forced oscillations in the linear combination. Since the natural (free) oscillations disappear as a result of damping in real systems in a relatively short time, it is worth looking for kinematic quantities pertaining to forced oscillations.

Assuming that this particular solution has the following form:

$$z = z_0 sin(\Omega t - \psi) \tag{1.62}$$

and after the introduction of

$$\vartheta = \frac{b}{b_{kr}} = \frac{\delta}{\Omega_0}, \tag{1.63}$$

and

$$\eta = \frac{\Omega}{\omega_0} \tag{1.64}$$

the deflection of forced oscillations of a body is:

$$\begin{aligned} z &= \frac{F_0}{c} \frac{1}{\sqrt{(1-\eta^2)^2 + 4\vartheta^2\eta^2}} sin(\Omega t - \psi) = \\ &= z_{st} \frac{1}{\sqrt{(1-\eta^2)^2 + 4\vartheta^2\eta^2}} sin(\Omega t - \psi), \end{aligned} \tag{1.65}$$

where z_{st} is the static deflection of a body due to the amplitude of the excitation force.

The phase displacement ψ of deflection z of a body and excitation force F is

$$\psi = arctg \frac{2\vartheta\eta}{1-\eta^2}. \tag{1.66}$$

The dynamic force F_U transmitted between an oscillating body and a frame may be obtained as the sum of the spring force and the damper:

$$F_U = b\dot{z} + cz = F_0 \sqrt{\frac{1 + 4\vartheta^2\eta^2}{(1-\eta^2)^2 + 4\vartheta^2\eta^2}} sin(\Omega t - \psi + \gamma). \tag{1.67}$$

The amplitude F_0 of excitation force F in relations (1.65) and (1.67) may be constant or excitation frequency-dependent Ω.

Where the excitation force F is generated as a result of unbalance, its amplitude is

$$F_0 = m_e r_e \Omega^2, \tag{1.68}$$

where m_e is the unbalanced mass and r_e is the radius of its circular orbit.

After substituting (1.68) into the relation (1.65) and (1.67), the amplitude z_0 of deflection z is in the form of:

$$z_0 = \frac{m_e r_e}{m} \frac{\eta^2}{\sqrt{(1-\eta^2)^2 + 4\vartheta^2\eta^2}} = z_e \frac{\eta^2}{\sqrt{(1-\eta^2)^2 + 4\vartheta^2\eta^2}}. \tag{1.69}$$

And the amplitude F_{U0} of the force F_U transmitted to the frame:

$$F_{U0} = \frac{m_e r_e}{m} c\eta^2 \sqrt{\frac{1 + 4\vartheta^2\eta^2}{(1-\eta^2)^2 + 4\vartheta^2\eta^2}} = F_e\eta^2 \sqrt{\frac{1 + 4\vartheta^2\eta^2}{(1-\eta^2)^2 + 4\vartheta^2\eta^2}}. \tag{1.70}$$

For a better overview of the amplitudes z_0 of deflections and the amplitudes F_{U0} of the forces transmitted to the frame, it is appropriate to introduce the ratios P_1 to P_4:

$$P_1 = \frac{z_0}{z_{st}}, P_2 = \frac{F_{U0}}{F_0}, P_3 = \frac{z_0}{z_e} \ a \ P_4 = \frac{F_{U0}}{F_e}. \tag{1.71}$$

Graphically shown are their waveforms depending on the ratio (1.64) of excitation frequency Ω and the natural frequency ω_0 on excitation frequency shown in *Fig. 1*.14.

The mathematical description of the oscillation of an excited dynamic system with one degree of freedom is supplemented with information about the phase displacement ψ of the excitation force F and the deflection z (*Fig. 1*.15*)*.

Fig. 1.14 shows that the deflection z of an oscillating object and the force F_U transmitted on the frame reaches its maximum value when the excitation frequency Ω is precisely equal to the natural frequency ω_0 of an undamped dynamic system. This condition is called "resonance" and the deflection z increases over time according to the relation

$$z = \frac{F_0}{2\Omega m} t sin\Omega t. \tag{1.72}$$

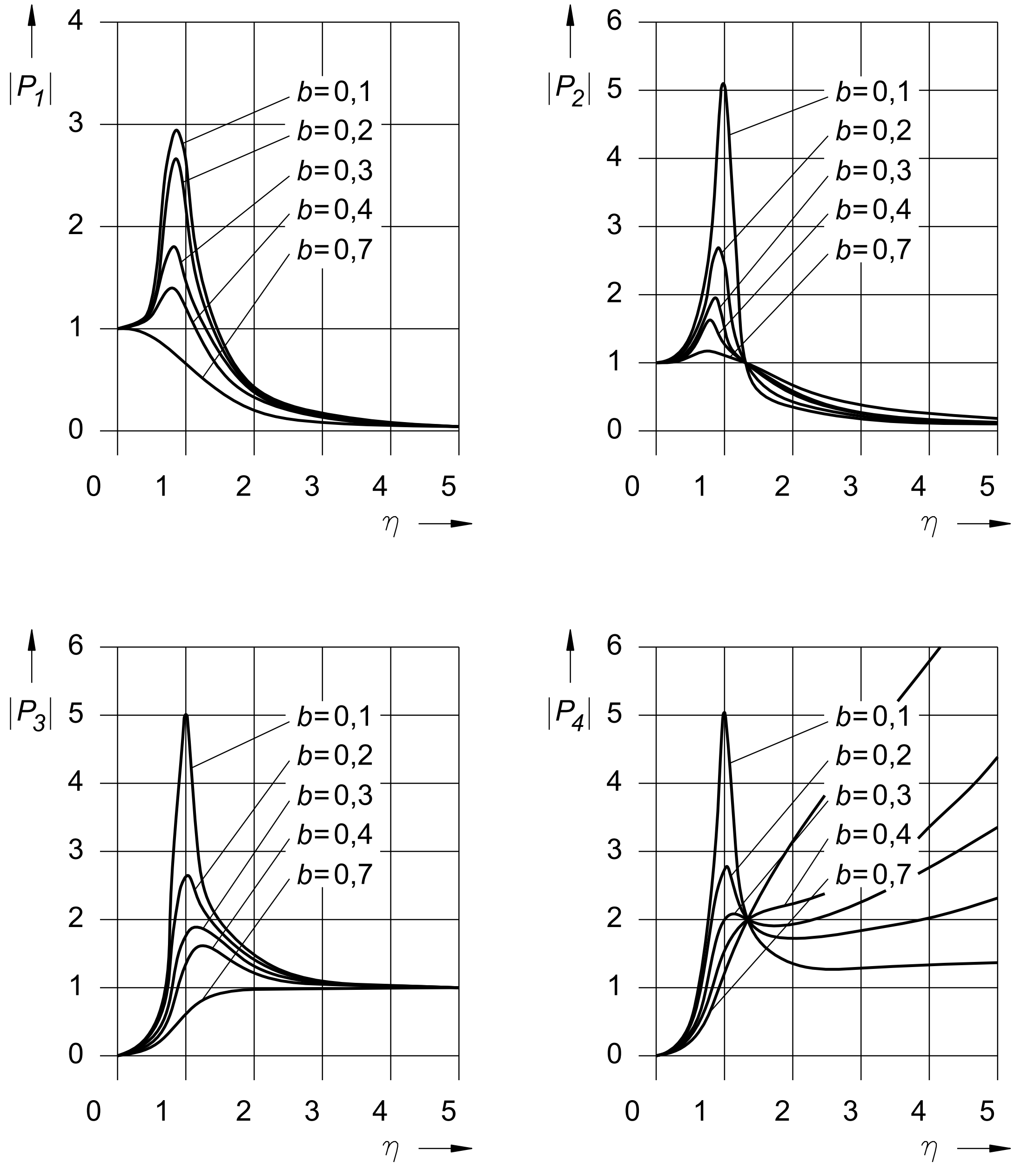


Fig. 1.14 Amplitude characteristics

The damping of a system is particularly important in resonance, because it allows to effectively reduce the amplitude of deflection z.

The amplitude of damped oscillations come from the relation (1.65) in the form of

$$z_0 = \frac{F_0}{b\Omega}. \tag{1.73}$$

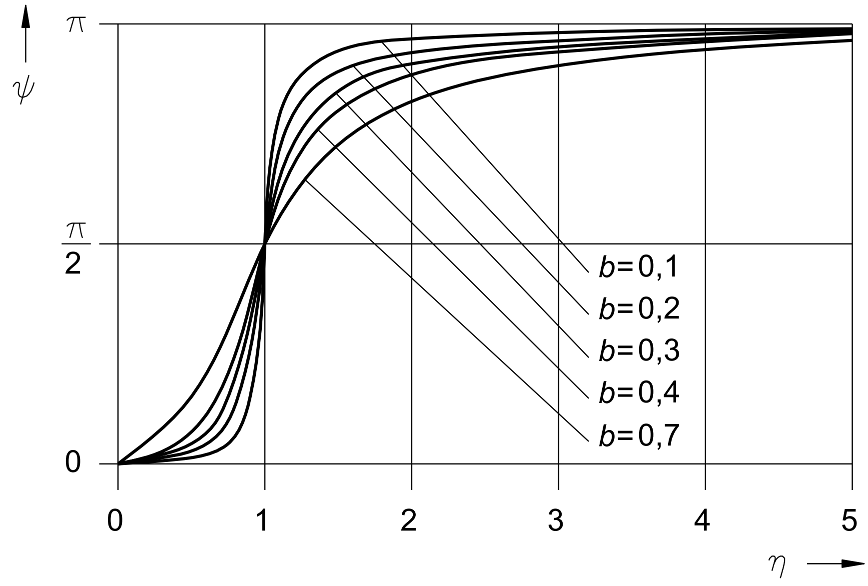


Fig. 1.15 Phase characteristics

The dynamics of machines have the knowledge about the natural frequency ω and thus the possibility of resonance paramount importance. Resonance may in no case be long. When passing through the resonance region, such as during start-up or braking, it is necessary to ensure sufficient output of engine and braking system to prevent the amplitude of oscillations from increasing dangerously.

The amplitude characteristics P_1 and P_2 (*Fig. 1*.14) show that the excitation frequency Ω in operation should be several times higher than the natural frequency ω. Under this condition, the amplitudes of deflection and the force transmitted to the frame are relatively small even without intensive damping.

The waveforms of amplitude characteristics P_3 and P_4 (*Fig. 1*.14) show that the amplitude of deflection in the non-resonance region depends on the value of $z_e(P_3 = 1)$ without the effect of damping. However, damping is of major importance to the forces transmitted to the base. Optimal state exists on condition when the excitation frequency Ω is about three times greater than the natural frequency ω and the system does not show a significant damping.

The behaviour of a dynamic system (*Fig. 1*.12b) with kinematic excitation, which is determined by deflection u of frame, can be described by differential equation

$$m\ddot{z} + b(\dot{z} - \dot{u}) + c(z - u) = 0, \tag{1.74}$$

where is m the weight, $z$the deflection of an oscillating object, b the damping coefficient, and c the rigidity of elastic connection between the object and frame.

The deflection u of the oscillatory motion of a frame can be assumed with respect to the*Fourier* series as harmonic

$$u = u_0 sin\Omega t \tag{1.75}$$

and is obtained from the equation

$$m\ddot{z} + b\dot{z} + cz = mu_0\sqrt{\omega_0^4 + (2\delta\Omega)^2} sin(\Omega t + \beta), \tag{1.76}$$

where β is the phase angle.

If monitoring of the motion of an object in time is introduced

$$\tau = t + \frac{\beta}{\Omega}, \tag{1.77}$$

The right side of the equation (1.76) will be as follows

$$mu_0\sqrt{\omega_0^4 + (2\delta\Omega)^2} = mu_0\omega_0^2\sqrt{1 + 4\vartheta^2\eta^2} = F_0. \tag{1.78}$$

The equation of motion (1.76) will be then in the same form as in (1.60) and the solution can be thus used.

After adjustment, the equation will be as follows:

$$z_0 = u_0\sqrt{\frac{1 + 4\vartheta^2\eta^2}{(1 - \eta^2)^2 + 4\vartheta^2\eta^2}} = u_0 P_2. \tag{1.79}$$

The phase displacement ψ of the deflection of an object z and a frame u is

$$\psi = arctg\frac{2\vartheta\eta^3}{1 - \eta^2 + 4\vartheta^2\eta^2}. \tag{1.80}$$

The phase characteristic in kinematic excitation is shown in *Fig. 1*.16.

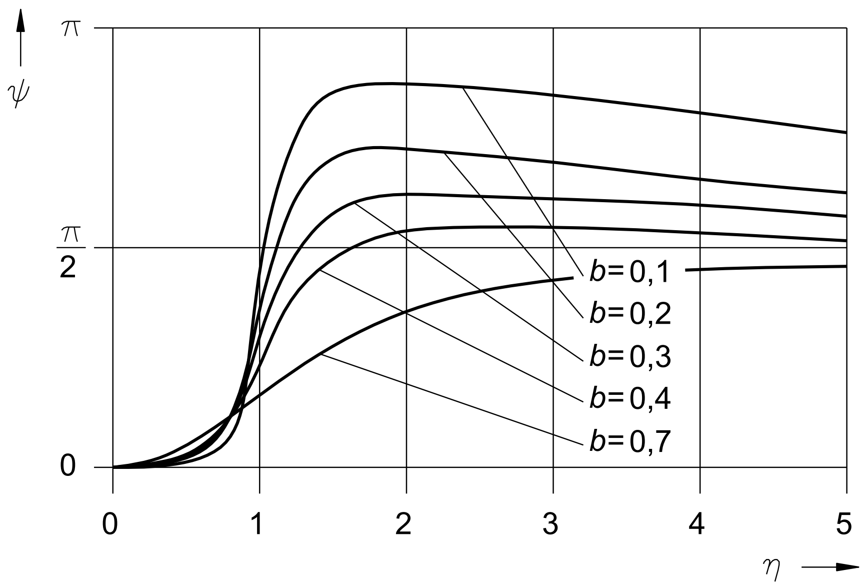


Fig. 1.16 Phase characteristic in kinematic excitation

The natural frequency ω of a kinematic-excited system should be, based on the waveform of the amplitude characteristic P_2 (*Fig. 1*.14) in operation, more than three times less than the excitation frequency Ω. Small damping leads to a further decrease in amplitude.

Like in systems with force excitation, the occurrence of resonance should be avoided.

Most dynamic systems, which are kinematic-excited, comes from automotive technology. Excitation is often characterised by randomness (stochastic waveform), so to minimize the amplitudes of deflections, additional dampers are required.

1.3.2 Single-mass System with Three Degrees of Freedom

A dynamic system with three degrees of freedom is a body with a plane of symmetry, in which there is the centre of gravity S and in which the force excitation acts simultaneously. The spring and possibly also the dampers are arranged symmetrically to this plane. Under this condition, the oscillating motion is a plane motion. This is the displacement in two coordinate axes and one rotation around an axis perpendicular to the plane of symmetry (*Fig. 1*.17).

From the perspective of machine dynamics, all natural frequencies of the system are very important. There are three values, namely for two displacements (in the y and z coordinates) and one rotation (around the x axis), which can be determined from the equations of motion for free undamped oscillations. According to the *d´Alembert's* principle, it will be as shown in (*Fig. 1*.17)

$$m\ddot{y} + c_{y1}y + c_{y2}y - \varphi c_{y1}a_z - \varphi c_{y2}a_z = 0, \tag{1.81}$$

$$m\ddot{z} + c_{z1}z + c_{z2}z - \varphi c_{z1}a_{y1} - \varphi c_{z2}a_{y2} = 0, \tag{1.82}$$

$$\begin{aligned} &J\ddot{\varphi} + c_{z1}za_{y1} + \varphi c_{z1}a_{y1}^2 - c_{y1}ya_z + \varphi c_{y1}a_z^2 + \varphi c_{y2}a_z^2 - \\ &-c_{y2}ya_z - c_{z2}za_{y2} + \varphi c_{z2}a_{y2}^2 = 0. \end{aligned} \tag{1.83}$$

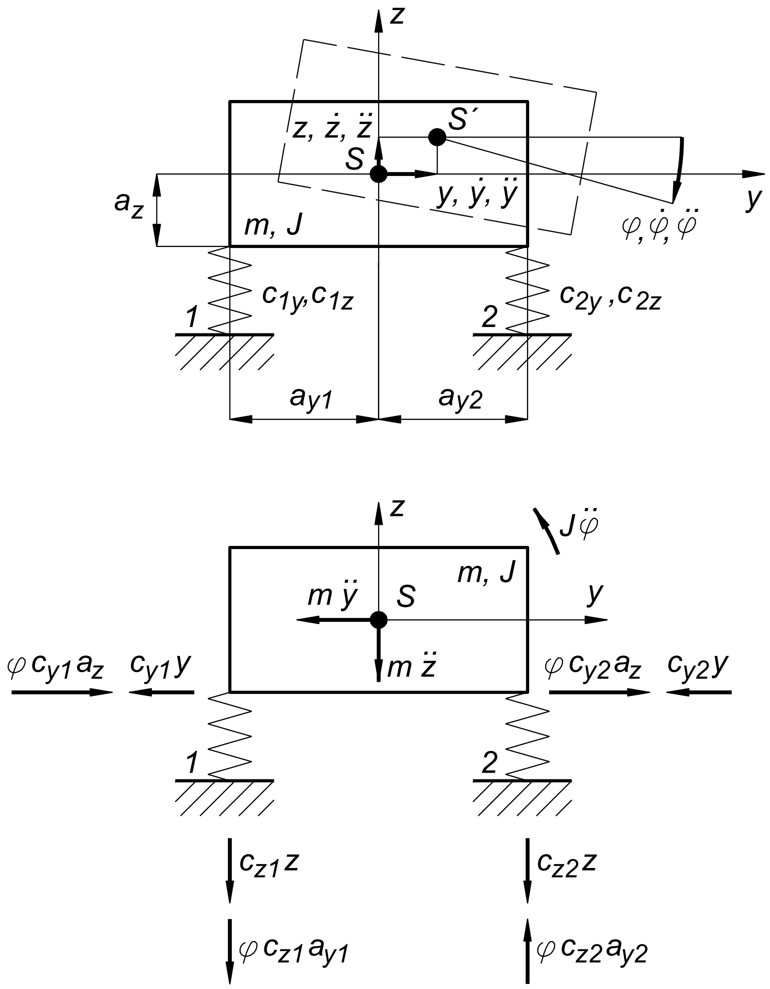


Fig. 1.17 Single-mass model with three degrees of freedom

The equations (1.81), (1.82) and (1.83) can be also written in matrix form:

$$\underline{M}\ddot{\vec{q}} + \underline{C}\,\vec{q} = \vec{0}. \tag{1.84}$$

Where

$$\underline{M} = \begin{bmatrix} m & 0 & 0 \\ 0 & m & 0 \\ 0 & 0 & J \end{bmatrix} \tag{1.85}$$

is the mass matrix,

$$\underline{C} = \begin{bmatrix} c_{y1} + c_{y2} & 0 & -a_z(c_{y1} + c_{y2}) \\ 0 & c_{z1} + c_{z2} & c_{z1}a_{y1} - c_{z2}a_{y2} \\ -a_z(c_{y1} + c_{y2}) & c_{z1}a_{y1} - c_{z2}a_{y2} & a_z^2(c_{y1} + c_{y2}) + (c_{z1}a_{y1}^2 + c_{z2}a_{y2}^2) \end{bmatrix} \tag{1.86}$$

is the stiffness matrix,

$$\ddot{\vec{q}} = \begin{bmatrix} \ddot{y} \\ \ddot{z} \\ \ddot{\varphi} \end{bmatrix} \tag{1.87}$$

is the acceleration vector, and

$$\vec{q} = \begin{bmatrix} y \\ z \\ \varphi \end{bmatrix} \tag{1.88}$$

is the displacement vector.

The solution will be performed similarly to the system with one degree of freedom. After substituting

$$\vec{q} = \vec{q}_0 sin(\omega t) \tag{1.89}$$

and after adjustment, it will be as follows

$$(\underline{C} - \omega^2 \underline{M})\vec{q}_0 = \vec{0}. \tag{1.90}$$

The amplitude $\vec{q}_0$ is obviously not equal to zero and therefore the determinant must be

$$Det|\underline{C} - \omega^2 \underline{M}| = 0. \tag{1.91}$$

The equation (1.91) gives three solutions for ω. These are three natural frequencies searched.

For forced undamped oscillations of a dynamic system with three degrees of freedom, the equation of motion will be in matrix form:

$$\underline{M}\ddot{\vec{q}} + \underline{C}\vec{q} = \vec{Q}. \tag{1.92}$$

Where is $\vec{Q}$ the vector of excitation force effects in the form of

$$\vec{Q} = \begin{bmatrix} F_{0y} \\ F_{0z} \\ M_{0x} \end{bmatrix} sin(\Omega t). \tag{1.93}$$

After substituting

$$\vec{q} = \vec{q}_0 sin(\Omega t) \tag{1.94}$$

it will be as follows

$$\left(-\Omega^2 \underline{M} + \underline{C}\right)\vec{q}_0 = \vec{Q}_0 \tag{1.95}$$

and gives the amplitude vector of deflection

$$\vec{q}_0 = \left(-\Omega^2 \underline{M} + \underline{C}\right)^{-1}\vec{Q}_0. \tag{1.96}$$

1.3.3 Multi-mass Dynamic Systems

In practice, there are many systems that consist of several oscillating bodies. They mostly perform the motions of the same nature, in one direction or around one axis of rotation. Excitation is often only on one body.

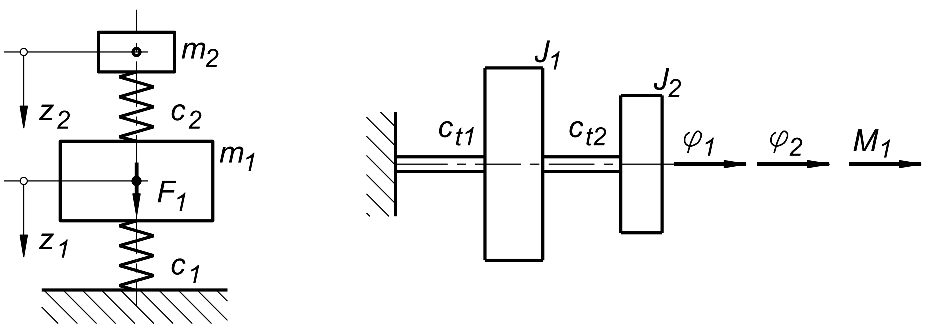
When considering a translational motion in the direction of the line, a dynamic system in the simple case can look like in *Fig. 1*.18a. These are two bodies with the masses m_1 and m_2, which are interconnected by a spring. One of the bodies is connected by means of another spring to the frame. To determine the position of the system, two figures and two coordinates z_1 and z_2 are required.

The equations of motion of free oscillations of both masses m_1 and m_2 according to the *d'Alembert* principle are

$$m_1\ddot{z}_1 + c_2(z_1 - z_2) + c_1 z_1 = 0, \tag{1.97}$$

$$m_2\ddot{z}_2 + c_2(z_2 - z_1) = 0. \tag{1.98}$$

The equations (1.97) and (1.98) contains the couplers $c_2 z_1$ and $c_2 z_2$.



a) b)

Fig. 1.18 Undamped dual-mass dynamic system:
a) translational motion, b) rotary motion

When substituting

$$z_1 = z_{10} sin(\omega t) \tag{1.99}$$

and

$$z_2 = z_{20} sin(\omega t) \tag{1.100}$$

we get

$$-m_1\omega^2 z_{10} + c_2(z_{10} - z_{20}) + c_1 z_{10} = 0, \tag{1.101}$$

$$-m_2\omega^2 z_{20} + c_2(z_{20} - z_{10}) = 0 \tag{1.102}$$

or in matrix notation:

$$\begin{vmatrix} c_1 + c_2 - m_1\omega^2 & -c_2 \\ -c_2 & c_2 - m_2\omega^2 \end{vmatrix} \begin{vmatrix} z_{10} \\ z_{20} \end{vmatrix} = 0. \tag{1.103}$$

If the amplitudes z_{10} and z_{20} should acquire final values, the determinant of the matrix in the equation (1.103) should be equal to zero. This gives two values for the natural frequencies

$$\omega_{1,2}^2 = \frac{1}{2}\left(\frac{c_1 + c_2}{m_1} + \frac{c_2}{m_2}\right) \pm \sqrt{\frac{1}{4}\left(\frac{c_1 + c_2}{m_1} + \frac{c_2}{m_2}\right)^2 - \frac{c_1 c_2}{m_1 m_2}}. \tag{1.104}$$

For excitation by harmonic force F_1 acting on body 1, the equations of motion of the system will be as follows:

$$m_1\ddot{z}_1 + c_2(z_1 - z_2) + c_1 z_1 = F_{10} sin(\Omega t), \tag{1.105}$$

$$m_2\ddot{z}_2 + c_2(z_2 - z_1) = 0. \tag{1.106}$$

When substituting

$$z_1 = z_{10} sin(\Omega t) \tag{1.107}$$

and

$$z_2 = z_{20} sin(\Omega t) \tag{1.108}$$

the amplitudes oscillations will be as follows

$$z_{10} = F_{10} \frac{c_2 - m_2 \Omega^2}{(c_1 - m_1 \Omega^2)(c_2 - m_2 \Omega^2) - c_2 m_2 \Omega^2}, \tag{1.109}$$

$$z_{20} = F_{10} \frac{c_2}{(c_1 - m_1 \Omega^2)(c_2 - m_2 \Omega^2) - c_2 m_2 \Omega^2}. \tag{1.110}$$

Derived relations apply in principle to torsional dynamic systems (*Fig. 1*.18b). The equations of motion of free oscillations are

$$J_1 \ddot{\varphi}_1 + c_{t2}(\varphi_1 - \varphi_2) + c_{t1}\varphi_1 = 0, \tag{1.111}$$

$$J_2 \ddot{\varphi}_2 + c_{t2}(\varphi_2 - \varphi_1) = 0 \tag{1.112}$$

and, analogously, harmonic deflections are expected in the following form

$$\varphi_1 = \varphi_{10} sin(\omega t), \tag{1.113}$$

$$\varphi_2 = \varphi_{20} sin(\omega t). \tag{1.114}$$

Substituting (1.113) and (1.114) into the equations (1.111) and (1.112) gives the following relations

$$-J_1 \omega^2 \varphi_{10} + c_{t2}(\varphi_{10} - \varphi_{20}) + c_{t1}\varphi_{10} = 0, \tag{1.115}$$

$$-J_2 \omega^2 \varphi_{20} + c_{t2}(\varphi_{20} - \varphi_{10}) = 0, \tag{1.116}$$

or

$$\begin{vmatrix} c_{t1} + c_{t2} & -c_{t2} \\ -c_{t2} & c_{t2} - J_2 \omega^2 \end{vmatrix} = 0. \tag{1.117}$$

This can be used to calculate two natural circular frequencies:

$$\omega_{1,2}^2 = \frac{1}{2}\left(\frac{c_{t1} + c_{t2}}{J_1} + \frac{c_{t2}}{J_2}\right) \pm \sqrt{\frac{1}{4}\left(\frac{c_{t1} + c_{t2}}{J_1} + \frac{c_{t2}}{J_2}\right)^2 - \frac{c_{t1} c_{t2}}{J_1 J_2}}. \tag{1.118}$$

For excitation by harmonic moment M_1 with an excitation frequency Ω acting on body 1, the equations of motion of the system will be as follows:

$$J_1 \ddot{\varphi}_1 + c_{t2}(\varphi_1 - \varphi_2) + c_{t1}\varphi_1 = M_{10} sin(\Omega t), \tag{1.119}$$

$$J_2 \ddot{\varphi}_2 + c_{t2}(\varphi_2 - \varphi_1) = 0 \tag{1.120}$$

When substituting

$$\varphi_1 = \varphi_{10} sin(\Omega t), \tag{1.121}$$

and

$$\varphi_2 = \varphi_{20} sin(\Omega t). \tag{1.122}$$

the amplitudes of angular deflection of oscillating motions of bodies will be as follows

$$\varphi_{10} = M_{10} \frac{c_{t2} - J_2\Omega^2}{(c_{t1} - J_1\Omega^2)(c_{t2} - J_2\Omega^2) - c_{t2}J_2\Omega^2}, \tag{1.123}$$

$$\varphi_{20} = M_{10} \frac{c_{t2}}{(c_{t1} - J_1\Omega^2)(c_{t2} - J_2\Omega^2) - c_{t2}J_2\Omega^2}. \tag{1.124}$$

Forced oscillation of a partially damped dynamic system is shown in *Fig. 1*.19 and can be described by the following differential equations:

$$m_1\ddot{z}_1 + b(\dot{z}_1 - \dot{z}_2) + c_2(z_1 - z_2) + c_1z_1 = F_{10}sin(\Omega t), \tag{1.125}$$

$$m_2\ddot{z}_2 + b(\dot{z}_2 - \dot{z}_1) + c_2(z_2 - z_1) = 0. \tag{1.126}$$

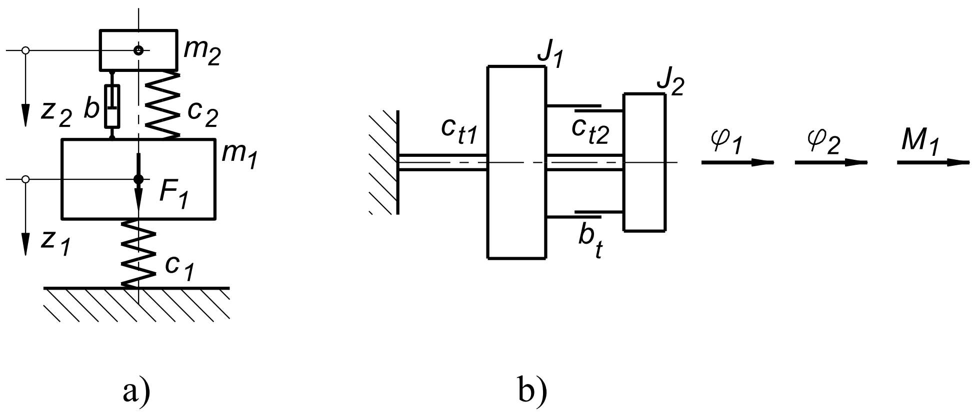


Fig. 1.19 Partially damped dual-mass dynamic systems: a) translational motion, b) rotary motion

Analogously, the following equations of motion apply to damped torsional systems (*Fig. 1*.19b):

$$J_1\ddot{\varphi}_1 + b_t(\dot{\varphi}_1 - \dot{\varphi}_2) + c_{t2}(\varphi_1 - \varphi_2) + c_{t1}\varphi_1 = M_{10}sin(\Omega t), \tag{1.127}$$

$$J_2\ddot{\varphi}_2 + b_t(\dot{\varphi}_2 - \dot{\varphi}_1) + c_{t2}(\varphi_2 - \varphi_1) = 0. \tag{1.128}$$

The amplitudes z_{10}, z_{20}, or φ_{10}, φ_{20} with small damping can be calculated from relations (1.109), (1.110), or (1.123), (1.124) without significant error.

The amplitudes z_{10} and φ_{10} of excited bodies with appreciable damping can be calculated from relations

$$z_{10} = F_{10}\sqrt{\frac{(c_2 - m_2\Omega^2)^2 + b^2\Omega^2}{[(c_1 - m_1\Omega^2)(c_2 - m_2\Omega^2) - c_2 m_2\Omega^2]^2 + b^2\Omega^2(c_1 - m_1\Omega^2 - m_2\Omega^2)^2}} \tag{1.129}$$

and

$$\varphi_{10} = M_{10}\sqrt{\frac{(c_{t2} - J_2\Omega^2)^2 + b_t^2\Omega^2}{[(c_{t1} - J_1\Omega^2)(c_{t2} - J_2\Omega^2) - c_{t2}J_2\Omega^2]^2 + b_t^2\Omega^2(c_{t1} - J_1\Omega^2 - J_2\Omega^2)^2}}. \tag{1.130}$$

For systems with several bodies, the calculation is substantially more difficult. However, in small damping, it is normally sufficient to determine the natural frequency of an undamped system. To determine the particular solution to oscillations, numerical methods can be advantageously used, where the parameters of springs and dampers may not necessarily be constant.

2 Minimization of Primary Dynamic Forces

The problems of machine dynamics can be effectively solved by minimizing the primary dynamic forces. These are the forces that arise due to uneven motion of machine elements. In mechanics, they are referred to as inertial forces.

If the mass m moves with acceleration a, there is a force acting on it according to the *Newton's* law with a magnitude of

$$F = ma. \tag{2.1}$$

While inertial forces in translational motion are difficult to minimize, in the rotating parts, there are many effective measures that allow to effectively solve this important role of the dynamics of machines.

In principle, the aim is to minimize the resultant forces, which are transmitted to the frame from the rotating parts. There are many methods that can be used for this purpose. Subsequently, some of them will be described.

2.1 Balancing of Rotors and Mechanisms

Especially desirable measures to minimize primary dynamic forces are primarily the structural solutions that do not bring too high production costs and do not assume special additional components. In practice, these measures are generally regarded as balancing.

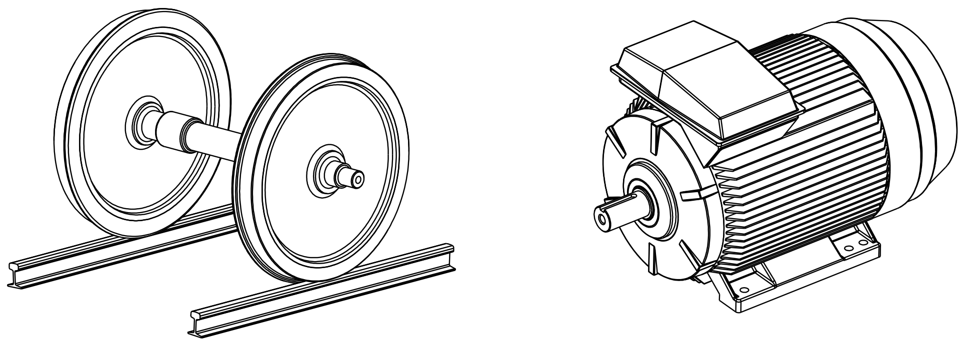


Fig. 2.1 Examples of rigid rotors

In some structures, balancing cannot be applied or its efficiency is too low. In these cases, dynamic absorbers or dampers are taken into account. In these structures, it is however necessary to take into account the deployment of additional special components.

2.1.1 Rotor Balancing

Balancing of rigid rotors (*Fig. 2.*1) represents the sub-role of overall balancing of mechanisms and is widely and successfully used in practice. It reduces adverse load of bearings and thus the structure as a whole.

Dynamic forces are generated in rotation of the rotor, which are transmitted to the bearings and from there to the machine frame and subsequently its base. The cause machine vibrations and additional loads, which can take a great intensity, because dynamic forces increase with the quadrate of the angular velocity.

The inertial forces of individual mass elements $\mathrm{d}m$ of a rotating body can be replaced by a resultant force $\vec{F}_D$ and resultant moment $\vec{M}_D$ in the selected origin of the coordinate system (*Fig. 2.*2).

If the body rotates around the axis $z \equiv \varsigma$ at an angular velocity $\vec{\omega}$ and an angular acceleration $\vec{\varepsilon}$, these vectors and their magnitudes can be calculated from the following relations:

$$\vec{\omega} = \vec{k} \times \omega \tag{2.2}$$

and

$$\vec{\varepsilon} = \vec{k} \times \varepsilon, \tag{2.3}$$

where $\vec{k}$ is the unit vector in the direction of coordinate z.

For acceleration $\vec{a}$, the following relation applies:

$$\vec{a} = \vec{\varepsilon} \times \vec{r} + \vec{\omega} \times (\vec{\omega} \times \vec{r}). \tag{2.4}$$

In relation (2.4), the first term and the second term on the right side of the equation represent tangential and normal acceleration, respectively.

The resultant inertial force will be

$$\vec{F}_D = \int_{(m)} d\vec{F}_D = \int_{(m)} -\vec{a}dm = -\vec{a}_s m. \tag{2.5}$$

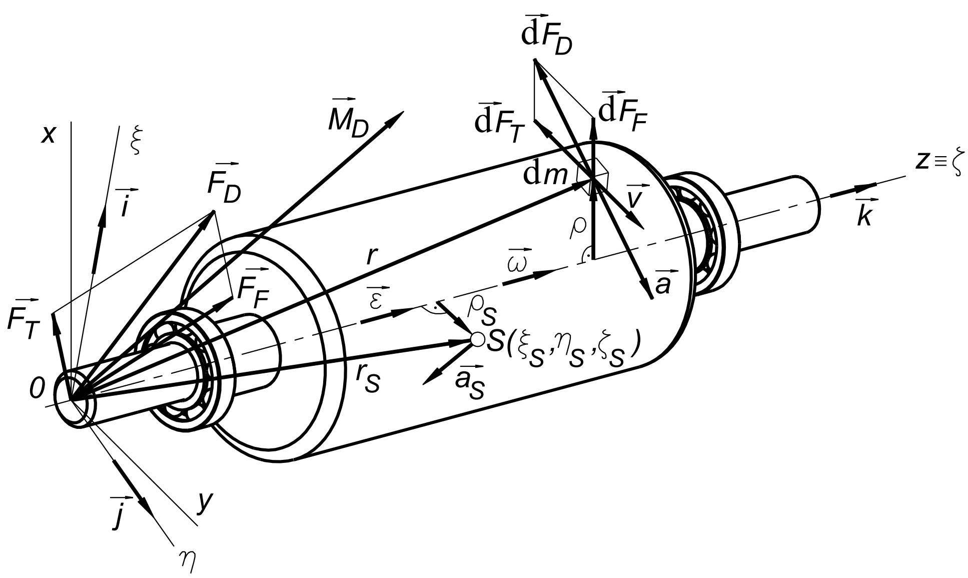


Fig. 2.2 Dynamic conditions during rotation

Its magnitude is determined by the weight m of a body concentrated in the centre of gravity S. But it acts in the origin of the coordinate system and, like the acceleration $\vec{a}$, it has two components, tangential force $\vec{F}_T$ and radial (centrifugal) $\vec{F}_F$ force. Then,

$$\vec{F}_D = \vec{F}_T + \vec{F}_F, \tag{2.6}$$

where

$$\vec{F}_T = m\vec{r}_S \times \vec{\varepsilon} = m\vec{\rho}_S \times \vec{\varepsilon} \tag{2.7}$$

and

$$\vec{F}_F = m\vec{\omega} \times (\vec{r}_S \times \vec{\omega}) = m\vec{\rho}_S \omega^2. \tag{2.8}$$

The resultant moment $\vec{M}_D$ to the origin of the coordinate system is

$$\vec{M}_D = \int_{(m)} \vec{r} \times d\vec{F}_D = \int_{(m)} \vec{a} \times \vec{r} dm = \\ = \int_{(m)} [\vec{r}(\vec{r}\vec{\varepsilon}) - \vec{\varepsilon}\vec{r}^2 + \vec{\omega} \times \vec{r}(\vec{\omega}\vec{r})] dm \tag{2.9}$$

and using the unit vectors in the directions of coordinate axes it can be written that

$$\vec{M}_D = \int_{(m)} \left[\vec{\imath}(\xi\zeta\varepsilon - \eta\zeta\omega^2) + \vec{\jmath}(\eta\zeta\varepsilon + \xi\zeta\omega^2) - \vec{k}(\xi^2 + \eta^2)\varepsilon\right] dm = \\ = \left(D_{\xi\zeta}\varepsilon - D_{\eta\zeta}\omega^2\right) + \vec{\jmath}\left(D_{\eta\zeta}\varepsilon + D_{\xi\zeta}\omega^2\right) - \vec{k}J_\zeta\varepsilon. \tag{2.10}$$

Its coordinates are

$$M_{D\xi} = \varepsilon D_{\xi\zeta} - \omega^2 D_{\eta\zeta}, \tag{2.11}$$

$$M_{D\eta} = \varepsilon D_{\eta\zeta} - \omega^2 D_{\xi\zeta}, \tag{2.12}$$

$$M_{D\zeta} = -\varepsilon J_\eta, \tag{2.13}$$

where $D_{\xi\zeta}, D_{\eta\zeta}$ are the products of inertia to the respective axes.

In well-balanced rotors, the inertial force $\vec{F}_D$ and the moment of inertia M_D must disappear at a uniform rotation ($\varepsilon = 0$). Therefore, the following conditions must be fulfilled:

$$\xi_S = 0, \tag{2.14}$$

$$\eta_S = 0, \tag{2.15}$$

$$D_{\xi\zeta} = 0 \tag{2.16}$$

and

$$D_{\eta\zeta} = 0. \tag{2.17}$$

The conditions (2.14) and (2.15) match static balancing and the conditions (2.16) and (2.17) match dynamic balancing of a rotating body.

The moment of inertia $M_{D\zeta}$ may not be equal to zero, because its effect does not load rotor bearings.

The role of rotor balancing can have three forms.

Fig. 2.3 shows three examples of unbalanced rotors. While Fig. 2.3a shows a purely static unbalance and Fig. 2.3b a purely dynamic unbalance, Fig. 2.3c shows a generally unbalanced rotor.

In the first case, the rotor thus shows a static unbalance, which can be described by means of the distance of the centre of gravity ρ_S from the axis of rotation $z \equiv \zeta$. It is a planar problem where the conditions (2.16) and (2.17) have already been fulfilled. Balancing is accomplished by adding or removing mass and thus the balancing force F_A in the plane of symmetry. The centre of gravity S is then on the axis of rotation, which corresponds to the conditions (2.14) and (2.15).

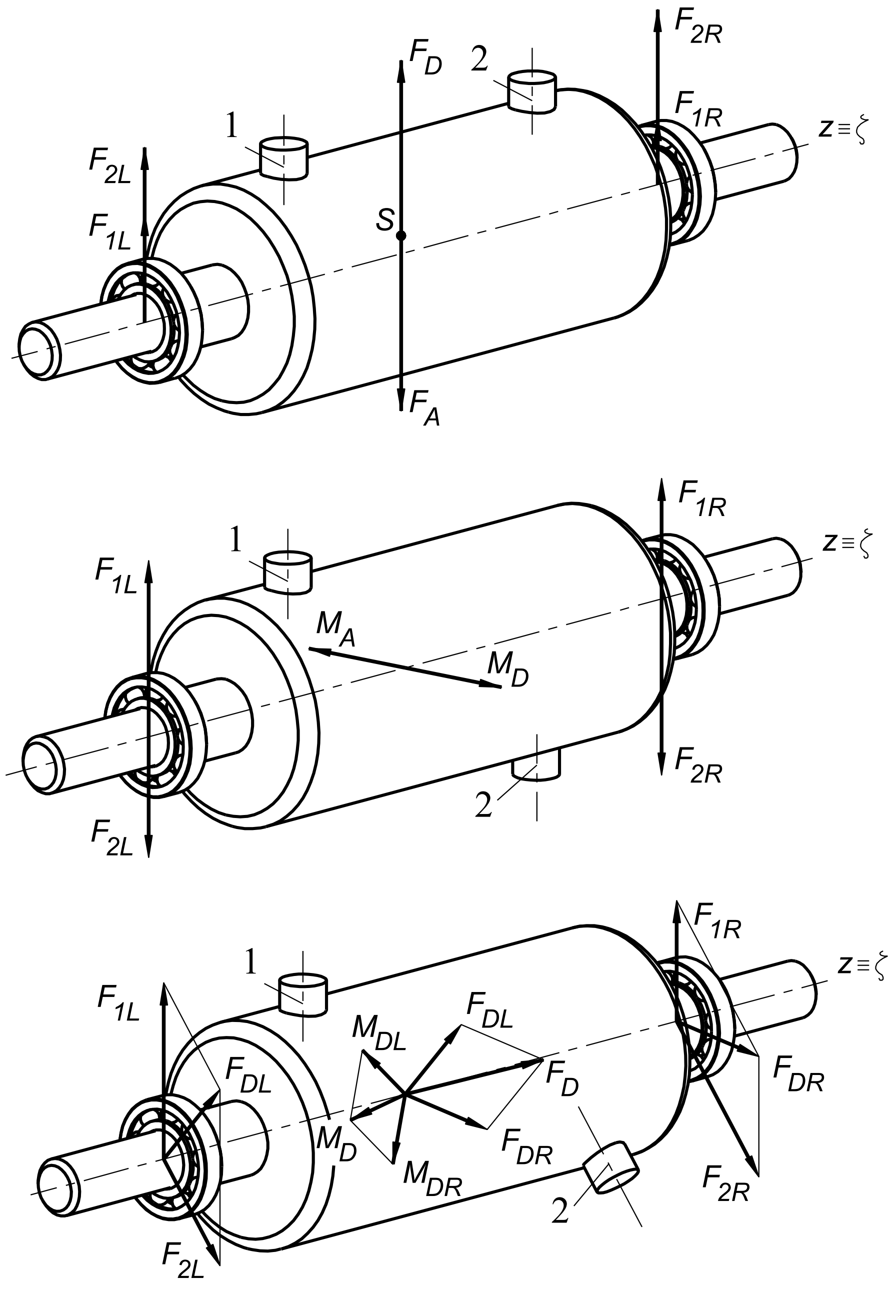


Fig. 2.3 Rotor balancing

In the second case, it is necessary to perform a purely dynamic balancing in two planes because the inertial forces of additional masses m_1 and m_2 must create an additional moment M_A. Then the conditions (2.16) and (2.17) can be met.

In *Fig. 2*.4, two additional masses m_1 and m_2 in two balancing planes of a rotating body, which are at a distance l_1 and l_2 from the selected point O on the axis of rotation $z \equiv \zeta$, are searched to balance a generally unbalanced rotor. To comply with the conditions (2.14), (2.15), (2.16) and (2.17), the following equations are available:

$$m\xi_S + m_1\xi_1 + m_2\xi_2 = 0, \tag{2.18}$$

$$m\eta_S + m_1\eta_1 + m_2\eta_2 = 0, \tag{2.19}$$

$$D_{\xi\zeta} + m_1\xi_1 l_1 + m_2\xi_2 l_2 = 0 \tag{2.20}$$

and

$$D_{\eta\zeta} + m_1\eta_1 l_1 + m_2\eta_2 l_2 = 0. \tag{2.21}$$

These are four equations for eight unknowns. Four of them can be therefore chosen. For practical reasons, the position of balancing planes, i.e. the values of l_1 and l_2, and the distances r_1 and r_2 of the balancing masses from the axis of rotation are chosen. To describe the position of the balancing masses, polar coordinates r_1, α_1 and r_2, α_2 are suitable. For correct balancing of the rotor, it is then necessary to examine the magnitude of additional masses m_1, m_2 and angles α_1 and α_2 defining their position, which are introduced into the calculation using the following relations:

$$\xi_1 = r_1 cos\alpha_1, \tag{2.22}$$

$$\eta_1 = r_1 sin\alpha_1, \tag{2.23}$$

$$\xi_2 = r_2 cos\alpha_2 \tag{2.24}$$

and

$$\eta_2 = r_2 sin\alpha_2. \tag{2.25}$$

Dynamically balanced bodies are balanced for any values of angular velocity ω and angular acceleration ε.

It should also be noted that the elastic rotors should be balanced in more than two planes and that such balancing is fulfilled only for a certain value of the angular velocity Ω.

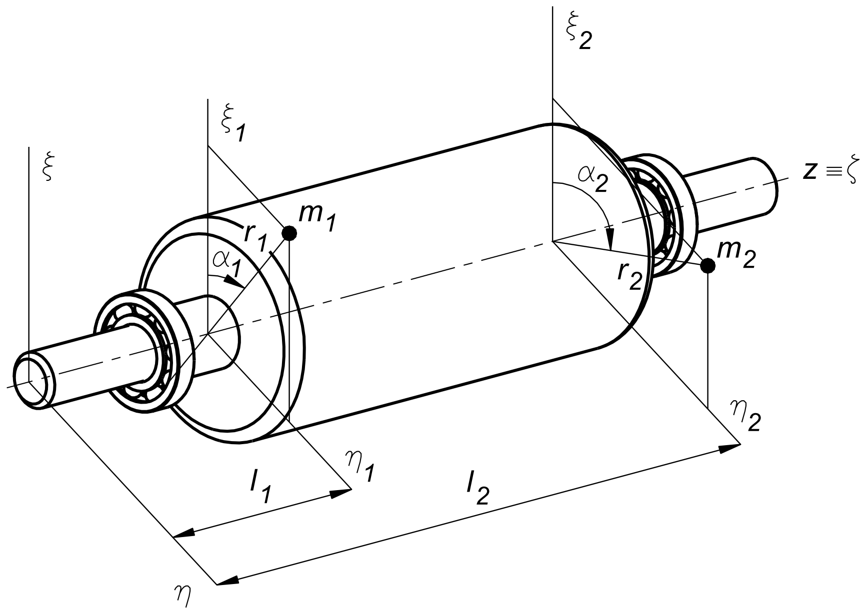


Fig. 2.4 Balancing in two planes

2.1.2 Balancing of Mechanisms

Balancing of a mechanism reduces the load on bearings, in which its components are connected to the machine frame. But it should be emphasised that the forces in joints and other movable joints of the mechanism itself can become larger and thus adversely affect the performance and life of the machine. Each action should therefore be considered carefully and a compromise should be identified between the advantages and disadvantages of balancing.

One balancing method frequently applied in practice uses an additional shaft with an eccentrically rotating mass (*Fig. 2*.5). Emerging centrifugal force counteracts the resultant inertial force of all members of a crank mechanism. In this case, the mechanism itself is not completely balanced, but it is important that its centre of gravity moves along the axis of symmetry.

The resultant inertial force has a periodic waveform with higher harmonics. Typically, the additional shaft is driven by the crankshaft through a gear set, the transmission of which corresponds to the dominant harmonic component. The example shown in *Fig. 2*.5 is the first harmonic.

Problems with balancing several mechanisms with common crankshaft are easier to solve due to possible suitable arrangement of individual mechanisms to each other than for a standalone mechanism. Due to this fact, multi-cylinder engines are used in the automotive industry, which are characterised by lower vibrations and noise.

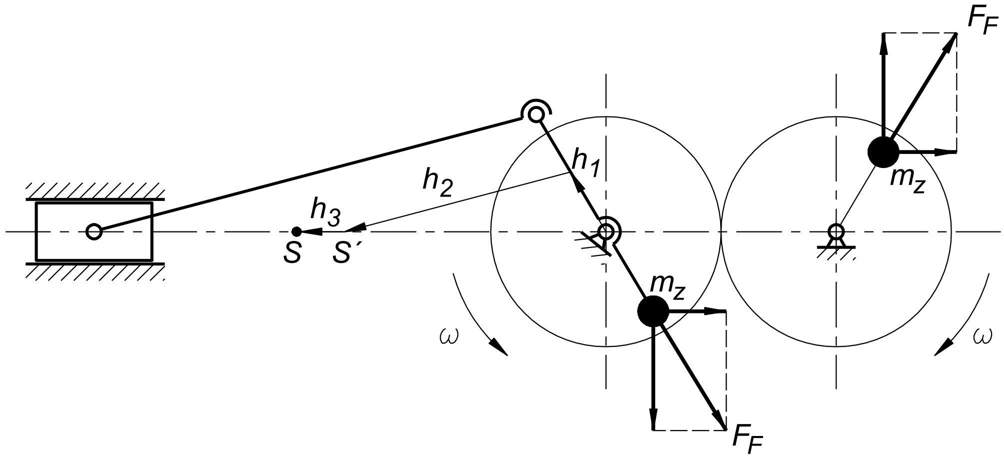


Fig. 2.5 Balancing by means of an additional shaft

2.2 Absorption of Vibrations

Reduction of the amplitude of deflection of a vibrating object can be realised by connecting a vibration absorber (*Fig. 2.*6). Then it is a dynamic system with two degrees of freedom, in which force or moment excitation acts on the object.

The condition for reducing the amplitude of deflection of an oscillating motion is that the natural frequency of the vibration absorber connected is equal to the excitation frequency. The absorption of vibrations can be realised in translational and rotary motions. Good functioning of the vibration absorber requires a purely harmonic waveform of excitation, otherwise its efficiency is low. Other requirements relate to requirements for very small damping.

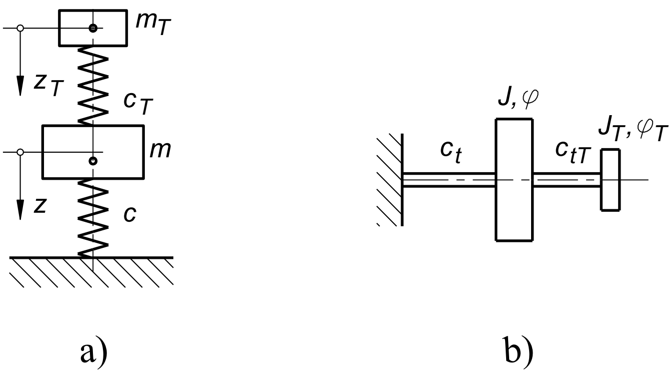


Fig. 2.6 Principle of the absorption of vibrations

2.2.1 Translational Oscillating Motion

When an object performs a translational oscillating motion, the amplitude of deflection can be reduced by means of a vibration absorber with a weight m_T and a stiffness c_T (*Fig. 2*.6a).

The equations of motion of this dynamic system with two degrees of freedom and the harmonic excitation force $F(\Omega t)$ acting on the object are

$$m\ddot{z} + c_T(z - z_T) + cz = F_0 sin(\Omega t), \tag{2.26}$$

$$m_T\ddot{z}_T + c_T(z_T - z) = 0. \tag{2.27}$$

The amplitudes of deflection z_0 of an object and an absorber z_{T0} are like (1.109) and (1.110),

$$z_0 = F_0 \frac{c_T - m_T\Omega^2}{(c - m\Omega^2)(c_T - m_T\Omega^2) - c_T m_T \Omega^2} \tag{2.28}$$

and

$$z_{T0} = F_0 \frac{c_T}{(c - m\Omega^2)(c_T - m_T\Omega^2) - c_T m_T \Omega^2} \tag{2.29}$$

When the denominator in equations (2.28) and (2.29) equals to zero, resonance arises. The equation

$$(c - m\Omega^2)(c_T - m_T\Omega^2) - c_T m_T \Omega^2 = 0 \tag{2.30}$$

can be used to calculate two roots. The results Ω_1 and Ω_2 correspond to resonant frequencies.

If the numerator in relation (2.28) equals to zero,

$$\Omega = \sqrt{\frac{c_T}{m_T}} = \omega_T \tag{2.31}$$

the amplitude z_0 of deflection of an object with a weight m equals to zero, and the vibration absorber oscillates only with a large but final amplitude, although no force is applied. Vibrations of an object disappear, but only at the excitation frequency that corresponds to the relation (2.31). This is an obvious disadvantage and therefore these systems are designed with dampers. However, damping affects adversely the minimization rate of vibrations (*Fig. 2*.7) and there are also higher energy losses.

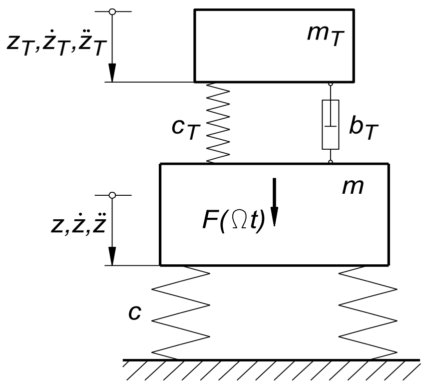


Fig. 2.7 Vibration absorber with a low damping

2.2.2 Rotary Oscillating Motion

In dynamic systems with a rotating object, there is also the possibility to minimize the amplitude of vibrations by means of a connected rotary vibration absorber (*Fig. 2*.6b). This is also a system with two degrees of freedom and with two resonance frequencies. The equations of motion are as follows

$$J\ddot{\varphi} + c_{tT}(\varphi - \varphi_T) + c_t\varphi = M_0 sin(\Omega t), \tag{2.32}$$

$$J_T\ddot{\varphi}_T + c_{tT}(\varphi_T - \varphi) = 0. \tag{2.33}$$

The mathematical model of this system is formally the same as in the previous paragraph. The results can thus be taken over when the masses m and m_T are replaced with the moments of inertia J and J_T, stiffness c and c_T torsional rigidities c_t and c_{tT}, and the excitation force $F(\Omega t)$ with the excitation moment $M(\Omega t)$.

In operating state, the excitation frequency Ω should be equal to the natural frequency ω_T of a vibration absorber (*Fig. 2*.8). However, it lies relatively tightly between the two resonance frequencies of the overall system and, therefore, this requirement should be met with sufficient accuracy. The above condition can be met more easily if the moment of inertia of a vibration absorber is greater. In this case, the resonance frequencies of the system have a greater interval.

It should be noted that the vibration absorber can be installed only in those drive systems, which have a constant or very little fluctuating operating frequency. As already stated, the lower resonance region must be quickly overcome in starting or stopping to avoid dangerous amplitudes of oscillations. For these reasons, it is necessary to provide the vibration absorber with damping to reduce the amplitudes in the area of resonance.

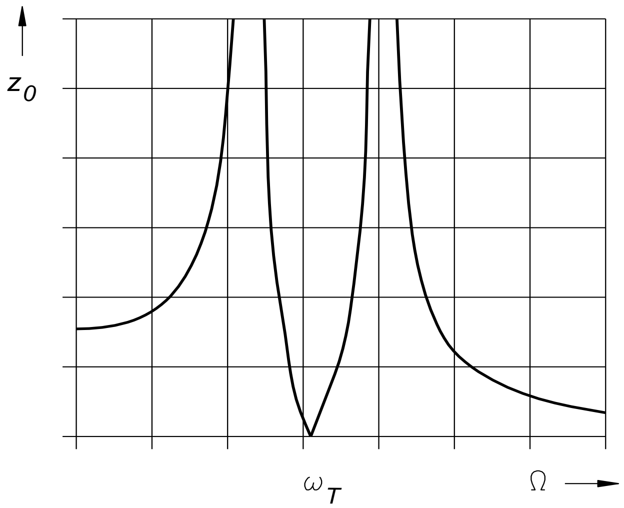


Fig. 2.8 *Waveform of the amplitude of oscillations of an object with a vibration absorber*

2.3 Oscillation Damping

Oscillation damping of an object can be realised by introducing damping into a dynamic system. The damper may be connected between the oscillating object and the frame or also between the object and another object. While in the first case, the number of degrees of freedom does not change, in the second case it increases by at least one.

Unlike vibration absorbers, the amplitudes of oscillations are reduced by damping on the basis of the conversion of mechanical energy into heat energy. However, energy losses are compensated due to the simple design, greater effect and relatively low production costs.

2.3.1 Translational Oscillating Motion

In dynamic systems where an object with a weight m performs a translational oscillating motion on a spring with a stiffness c, the amplitudes of deflection z_0 of an object are minimized with the use of dampers with a coefficient of damping b connected to the frame (*Fig. 1*.12a). These measures are effective and often used in practice and offer many variants in structural design, where there are systems with one degree of freedom.

If appropriate, damped systems are used, where a body with a mass m_D is connected to an oscillating object by means of a spring with a stiffness c_D and a damper with a damping coefficient b_D. It is a dynamic system with two degrees of freedom and, in principle, it is a vibration absorber with additional considerable damping b_D (*Fig. 2*.7).

The equations of motion

$$m\ddot{z} + b_D(\dot{z} - \dot{z}_D) + c_D(z - z_D) + cz = F_0 sin(\Omega t), \tag{2.34}$$

$$m_D\ddot{z}_D + b_D(\dot{z}_D - \dot{z}) + c_D(z_D - z) = 0 \tag{2.35}$$

give the dependence of the amplitude of deflection of an object on the excitation frequency:

$$z_0 = F_0 \sqrt{\frac{(c_D - m_D\Omega^2)^2 + b_D^2\Omega^2}{[(c - m\Omega^2)(c_D - m_D\Omega^2) - c_D m_D \Omega^2]^2 + +b_D^2\Omega^2[c - m\Omega^2 - m_D\Omega^2]^2}}. \tag{2.36}$$

For optimal tuning, the relation (2.36) should be strictly respected in the design.

2.3.2 Rotary Oscillating Motion

Damping an oscillating motion of a rotating object with a moment of inertia J excludes apparently connection of the damper to the frame. So there is only the variant, in which a damper with a damping coefficient b_{tD} is connected to an object and to an additional co-rotating body with a moment of inertia J_D. The connection of both bodies is usually supplemented with a spring with a stiffness c_{tD} (*Fig. 2.9*).

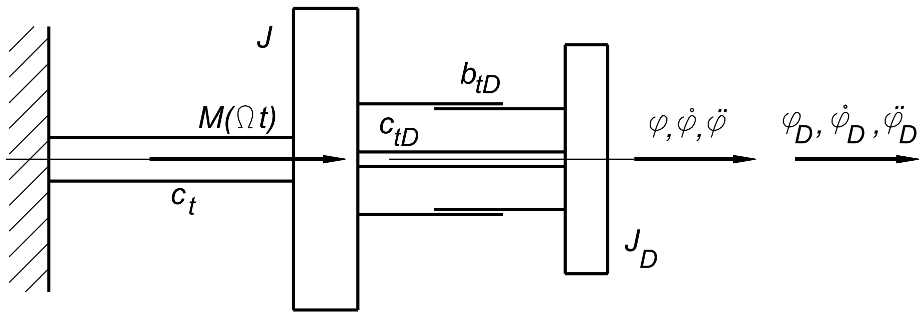


Fig. 2.9 Torsional vibration damper

The equations of motion for an object, on which a harmonic excitation moment acts, are

$$J\ddot{\varphi} + b_{tD}(\dot{\varphi} - \dot{\varphi}_D) + c_{tD}(\varphi - \varphi_D) + c_t\varphi = M_0 sin(\Omega t), \tag{2.37}$$

$$J_D\ddot{\varphi}_D + b_{tD}(\dot{\varphi}_D - \dot{\varphi}) + c_{tD}(\varphi_D - \varphi) = 0. \tag{2.38}$$

Since the connection to a frame has only small damping, the equations (2.37) and (2.38) take into account only damping b_{tD}.

For the amplitude of deflection φ_0 of oscillation in a rotary motion, the equations (2.37) and (2.38) give

$$\varphi_0 = M_0 \sqrt{\frac{(c_{tD} - J_D\Omega^2)^2 + b_D^2\Omega^2}{[(c_t - J\Omega^2)(c_{tD} - J_D\Omega^2) - c_{tD}J_D\Omega^2]^2 + b_D^2\Omega^2[c_t - J\Omega^2 - J_D\Omega^2]^2}}. \tag{2.39}$$

The waveform φ_0 depending on the excitation frequency Ω is shown in *Fig. 2*.10.

At zero damping, there are two natural frequencies. Should damping be infinitely large, the system shows only one natural frequency. For all other values of damping, the amplitude characteristics always pass through two intersections P and Q (*Fig. 2*.10). Optimal tuning of a system depends on the waveform, in which the amplitudes at the points P and Q are the same and tangents at these points are horizontal.

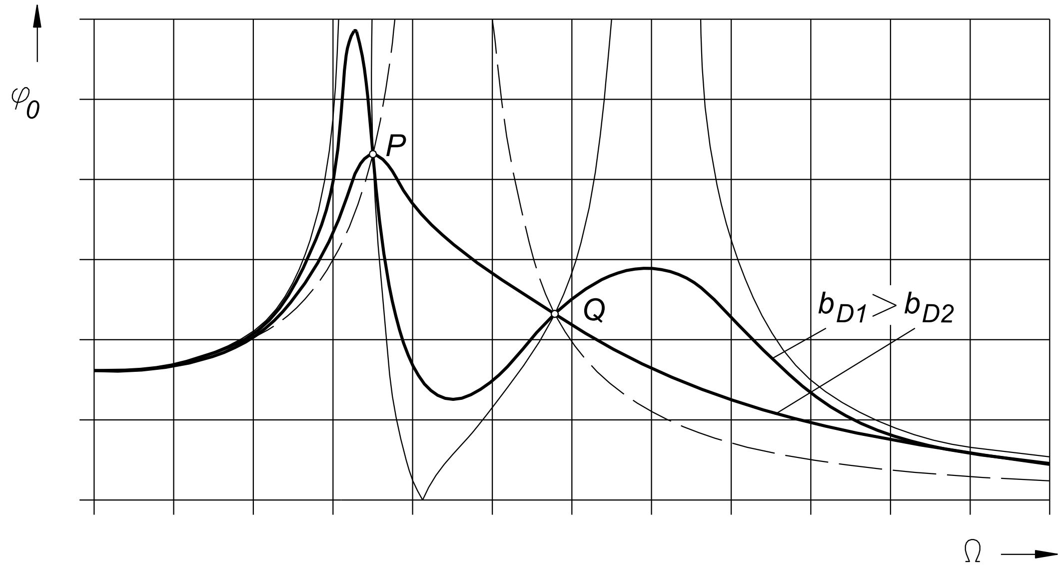


Fig. 2.10 Amplitude characteristic of a damped rotor

Examples of practical design of dampers of torsional vibrations are shown in *Fig.* 2.11.

In practice, there are dampers without further elastic coupling to a rotating body; their inertial mass is only connected through a damping fluid to an oscillating object. In the computational model (2.37) and (2.38) as well as in the relation (2.39), a zero value of the torsional rigidity c_{tD} of a damper will be substituted. However, it should be noted that there is a considerable springing effect in damping fluids with higher viscosity. Therefore, the following results are limited by this fact. *Fig. 2*.12 shows functional dependences of the amplitude of oscillations φ_0 on the excitation frequency Ω.

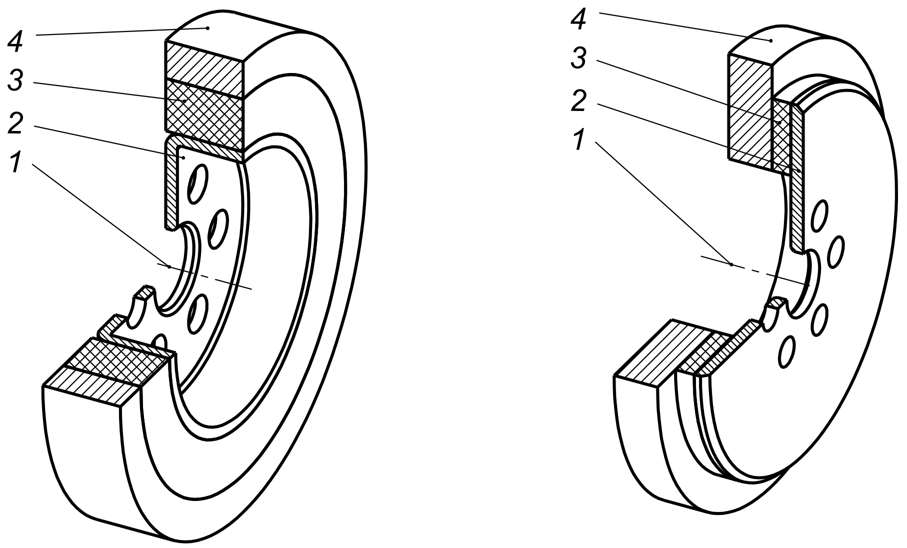


Fig. 2.11 Rotary damper: 1-axis of rotation, 2-disk, 3-rubber spring, 4-damping mass

At zero damping b_{tD}, this characteristic corresponds to an undamped system with a moment of inertia J and a torsional rigidity c_t and the damper has no effect. At infinite value of damping b_{tD}, the moment of inertia J increases by the value J_D of the damper. Resonance then occurs at a lower excitation frequency.

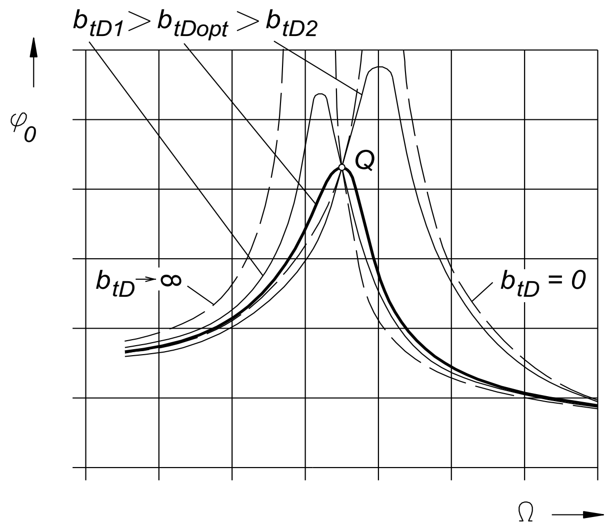


Fig. 2.12 Amplitude of oscillations of a damped rotor with a damper without elastic coupling

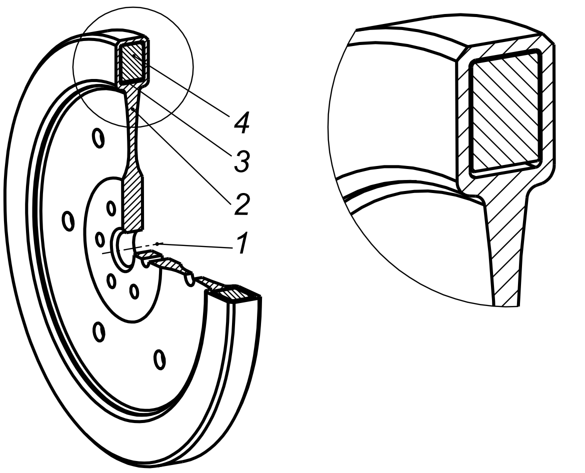


Fig.2.13 Design of damped rotor with a damper without elastic coupling: 1-axis of rotation, 2-housing, 3-damping fluid, 4-inertial mass

The amplitude function with a maximum at the point where both characteristics intersect is the optimal state of the damping of a system.

The rotary dampers without the elastic coupling are provided with a viscous fluid or a friction.

A rotary damper with a viscous fluid (*Fig.2.*13) consists of the housing connected to the shaft, which accommodates the inertial mass and the oil filling. In practice, silicone oils are used as the damping oils. Dampers of this design are applied in large diesel engines and can have a diameter up to two meters.

Friction dampers have an inertial mass which is frictionally coupled to a connecting disc and subsequently to a damped system. Brake linings are used on the contact surfaces and the required friction moment is set by pre-loading the springs. The inertial mass of a damper is connected to a dampened system in a rotary manner by means of plain (sliding) bearings.

3 Minimization of Vibration Transmission

Almost every machine in operation is a source of dynamic forces that are transmitted into the ground. This produces unfavourable vibrations, which cause dynamic stress in the foundations and the entire structure of a building. This leads to the fact that not only designers and manufacturers of machines, but also the operators must be interested in dynamic behaviour of machines. Therefore, this issue is one of the most important issues in the dynamics of machines. All measures to minimize the adverse vibrations generated in operation of machines are commonly called "vibration isolation".

Minimization of vibration transmission applies also to aggregates, which are operated in a frame, body, etc., and simultaneously cause their dynamic loading.

Each machine base shows greater or lesser elastic properties, thus affecting immediately the dynamic behaviour of the entire dynamic system.

In practice, there are different designs. Foundation blocks of considerable weights are often used to reduce the transmission of vibrations. Concrete foundation is typically placed in natural soil (*Fig. 3*.1).

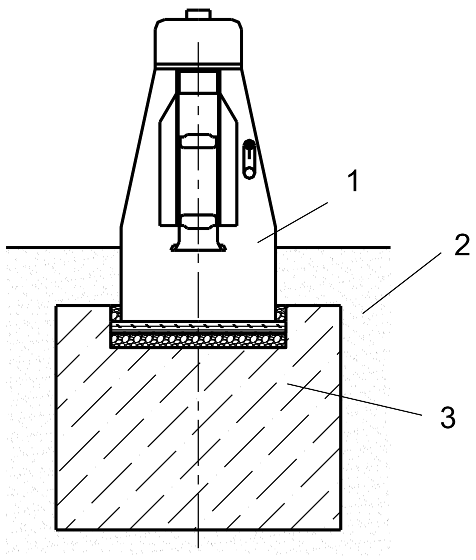


Fig. 3.1 Mounting of the machine on a concrete foundation: 1-machine, 2-natural soil, 3-concrete foundation

For setting a machine with rotating parts, various frame structures (*Fig. 3*.2) are designed, which exhibit the appropriate springing characteristics in the horizontal direction.

In addition to these methods of setting a machine, flexible foundations are dealt with in the field of machine dynamics (*Fig. 3*.3). Their structure usually consists of a frame, to which steel, pneumatic or rubber springs are attached. Where the static load changes significantly in operation, air springs are used that allow relatively easy adjustment and setting of the static height only by changing air overpressure.

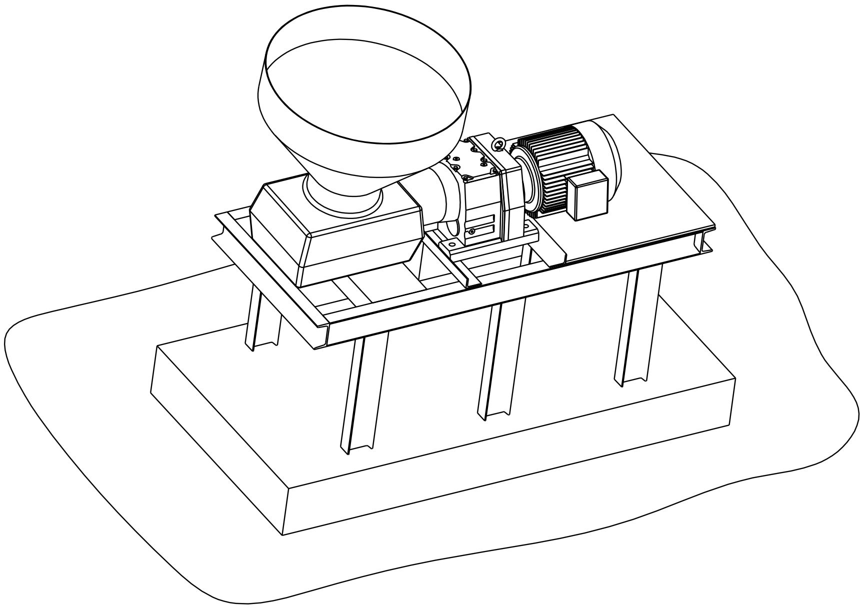


Fig. 3.2 Setting of machine onto a frame structure

The principle of flexible foundation is illustrated schematically in *Fig. 3*.3. In this case, the design represents the choice of springs and dampers, and the determination of optimal parameters. These data as well as the position and location of the springs and dampers are introduced into the optimisation calculation of a dynamic system. The parameters of a flexibly mounted machine, weight, moments of inertia, force excitation and the excitation frequency play also an important role.

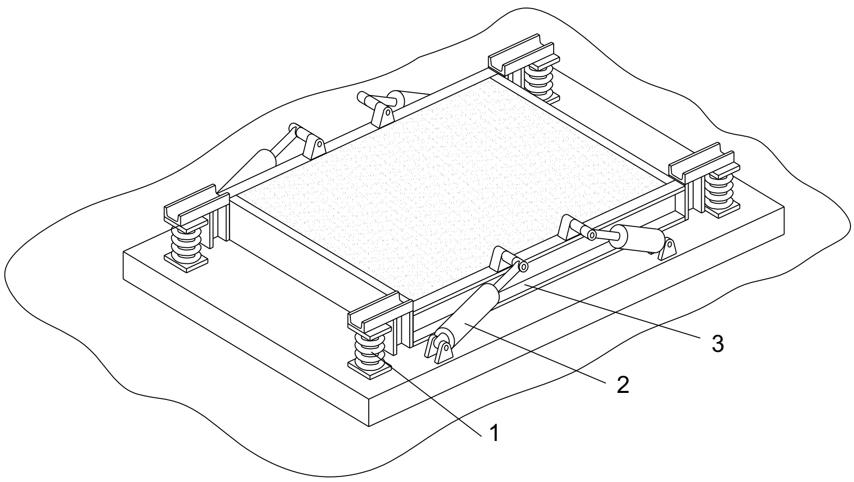


Fig. 3.3 Flexible machine base: 1-springs, 2-dampers, 3-frame.

The main criteria in the design of parameters of springs and dampers, stiffness and damping coefficients are the magnitudes of the forces transmitted to the ground and the deflections of an oscillating motion of a machine. These two criteria are often in conflict, so the designer must find a suitable compromise.

The roles of vibration isolation also apply to devices that are exposed to the effect of vibrations from the surroundings. A typical example is the whole area of vehicles where vibration isolation of persons, e.g. driver's seat (*Fig. 3.*4), plays an important role. Similar problems represent mounting systems for measuring and computing technology.

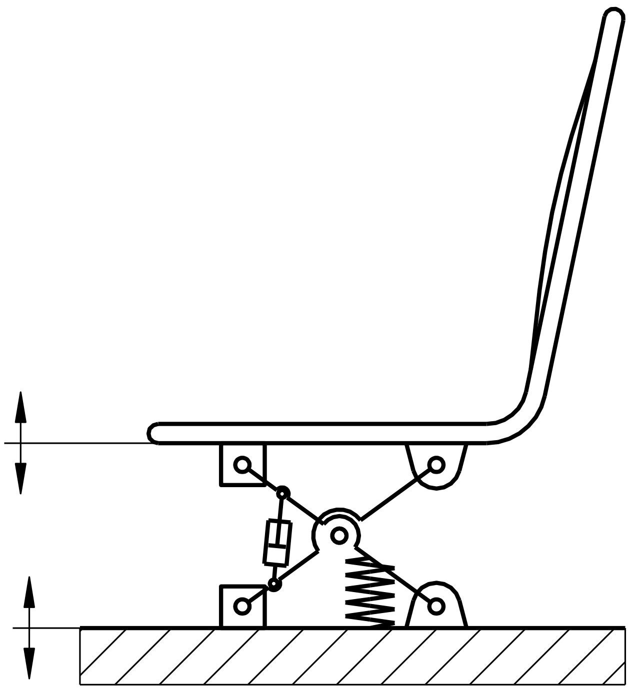


Fig. 3.4 Suspension driver's seat with a guide mechanism

Possibilities of using the computational models referred to in the first chapter will be applied to the representative cases of vibration isolation systems. Machines and equipment will be grouped by the type and nature of the excitation effects.

3.1 Machines with Periodic Force Excitation

Periodic excitation arises in machines, where the working process repeats on a regular basis and at short intervals. Moving parts of functional mechanisms perform generally a rotary motion. The more complicated are the motions of components, the more diverse inertial forces are generated as the basis of excitation effects. Flexible mounting of a weaving machine is subsequently shown as a representative example.

Vibration and noise related problems in weaving mills are generally known. Intensive dynamic forces generated during the operation of weaving machines are transmitted into the ground and cause adverse vibrations (*Fig. 3*.5). For these reasons, the weaving machines are mostly operated on the ground floor of the production hall, where the effects of dynamic forces are relatively small. Completely different situation arises when the machines are located on a higher floor in a building. In this case, vibration isolation measures take a great importance.

Fig. 3.5 Weaving machine

In practice, there is a large variety of flexible mountings, which are installed between the weaving machine and the floor in order to reduce the transmission of dynamic forces. One of the designs is shown below.

The design of a weaving machine shows two lateral parts, which are connected by means of a cross member and in which relatively long shafts are mounted. Torsional rigidity depends primarily on cross members connecting the two lateral parts of a machine and should be large enough to prevent damage or reduced life of bearings due to torsional deformations. Thrifty manufacturers often dimension machine frames with low torsional rigidity and assume that the reinforcement occurs after attaching the machine to the floor. For these reasons, it is necessary to consider possible reinforcement of frame structure in the design of flexible mounting.

The design, which has proved itself in many practical cases, consists in dividing elastic elements into a group of low rigidity and a group of multiple higher rigidity. The first group can be realised by means of pneumatic springs and the other group by means of rubber springs. Because the excitation forces arise in particular in the front of the weaving machine, supporting with relatively soft springs is preferable. A greater part of its mass is concentrated at the rear of the machine and supporting with relatively rigid rubber springs (*Fig. 3*.6) is preferable here. The aim of this solution is to increase the torsional rigidity of a machine.

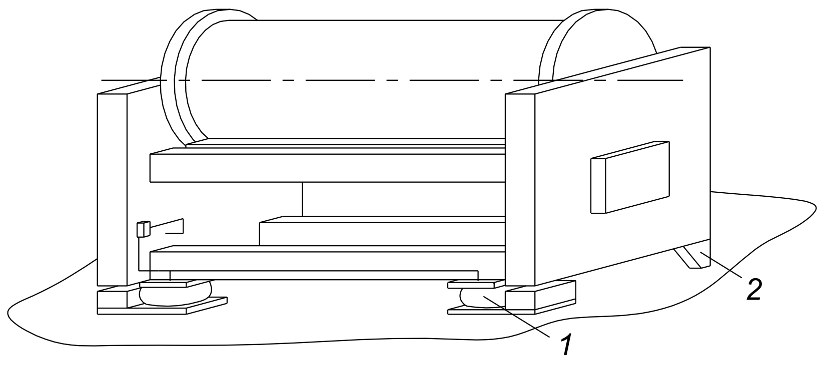


Fig. 3.6 Flexible mounting of the loom: 1-pneumatic spring, 2-rubber spring

Most weaving machines has a vertical plane of symmetry in the longitudinal direction and the computational model for a single-mass system with three degrees of freedom can therefore be used to describe the dynamic behaviour (*Fig. 3*.6).

Excitation forces of a weaving machine are periodic but show no harmonic character. The fundamental harmonic component is the first one, which belongs to the machine speed. The natural frequency of the system must therefore be at least three times less than the excitation frequency to achieve a satisfactory efficiency of vibration isolation.

3.2 Machines with Impact Force Excitation

Machines, which show an impact force excitation in operation, include undoubtedly forging hammers (*Fig. 3*.7). Their flexible mounting is of great importance, since the dynamic effects of these machines are very intensive.

The initial conditions to develop a computational model involve the fact that the machine has two vertical planes of symmetry. The choice of symmetric placement of springs and dampers is then efficient. Due to the large magnitude of excitation forces, relatively large amplitudes of deflection of an oscillating motion of a machine should be considered. Since the impact force acts in the vertical axis defined by the intersection of both planes of symmetry, the computational model of a single-mass system with one degree of freedom can be introduced. Wave pneumatic springs and hydraulic telescopic dampers can be considered for the design (*Fig. 3*.8).

Fig. 3.7 Forging hammer

After impact, the machine as a whole receives energy from the ram and thus the initial speed $\dot{z}(0) = v_0$ and then will produce a free damped oscillation.

When calculating the maximum deflection z_0 of the oscillating motion, the equation of motion (3.1) can be used. Because the oscillating motion will have relatively large amplitudes of deflection, non-linear springing and damping characteristics of pneumatic springs and hydraulic telescopic dampers should be introduced in the computational model. The equation of motion will be as follows:

$$m\ddot{z} + jF_D(\dot{z}) + iF_F(z) = 0, \tag{3.1}$$

where m is the total weight of the forging hammer, j is the number of dampers, $F_D(\dot{z})$ is the damping force, i is the number of springs and $F_F(z)$ is the restoring force.

The initial conditions for the solution to the equation (3.1) are $z(0) = 0$ and $\dot{z}(0) = v_0$. The initial speed v_0 of forging hammer with a restitution coefficient equal to 1 (perfectly elastic body) comes from the equation:

$$v_0 = 2\frac{m_S v_{S0}}{m_S + m}, \tag{3.2}$$

where m_S is the weight of the ram and v_{S0} is its speed just before the impact.

The speed v_{S0} of the ram can be calculated from the equation

$$E_S = \frac{1}{2} m_S v_{S0}^2, \tag{3.3}$$

because the kinetic energy E_S of the ram is usually relatively accurately stated by the manufacturer.

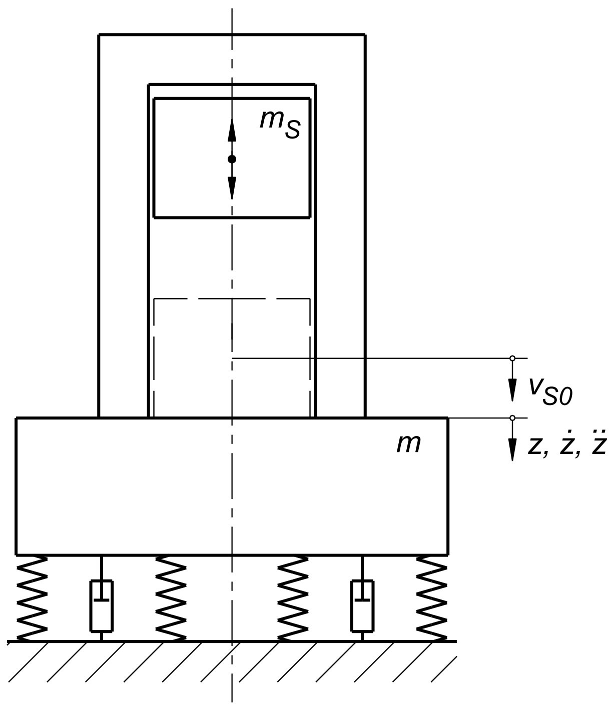


Fig. 3.8 Mechanical model of the spring-mounted forging hammer

3.3 Machines with Kinematic Excitation

Kinematic excitation occurs very often in vehicles where not only aggregates and other components but also persons are exposed to adverse vibrations. For that reason, an important field was established in engineering practice that deals with this issue. Besides solving springing and damping of frames and vehicle bodies, reducing vibrations in the driver's seat is one of the very important tasks in the automotive industry (*Fig. 3.9*).

Fig. 3.9 Driver's seat

Kinematic excitation of the driver's seat is caused by road surface irregularities and has a stochastic nature. For that reason, the natural frequency of the dynamic system of the seat with driver should be very small to ensure a sufficient reduction of the transmission of all vibration components. Limitation of the value of natural frequency is linked to ensuring a certain operation of the controls of the vehicle. In guide mechanism of seat pedestal, the spring acts generally in parallel with a damper.

The dynamic system of the driver's seat matches a single-mass model with one degree of freedom because one coordinate is enough to describe the position of the guide mechanism. Calculation of the amplitude of deflection of the oscillating motion must contain the damping coefficient and is limited to the theoretical conditions of periodic excitation. Laboratory simulations are used to set optimal springing and damping of the pedestal under real conditions of excitation. Calculation of the oscillating motion is then used to set default parameters and assess their impact on the behaviour of the dynamic system.

Because there are rods of different nature, there is a requirement for setting or regulating the natural frequency of the seat with respect to the nature of excitation. This is achieved with the use of the structure shown in *Fig. 3*.10. These are two counteracting pneumatic springs, one of which raises the seat and the other lowers the seat. This concept can change the rigidity of the pedestal independently of driver's weight and therefore the natural frequency.

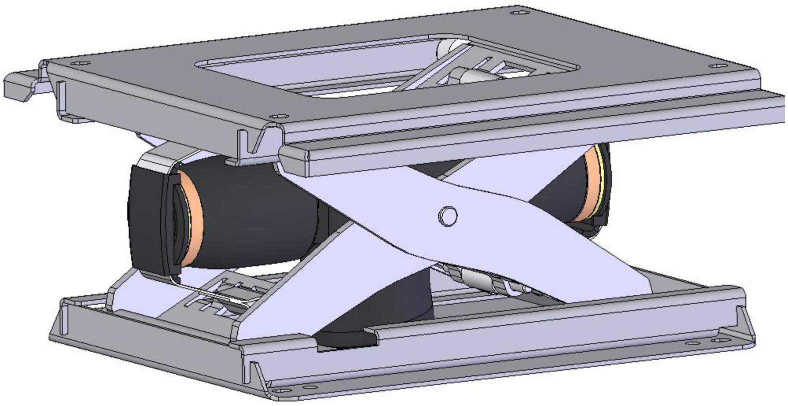


Fig. 3.10 *Pneumatic pedestal of driver's seat with adjustable natural frequency*

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